

The Trigonometric Identities

The Fundamental Identities	
Reciprocal Identities	
$\sin x = \frac{1}{\csc x}$	$\csc x = \frac{1}{\sin x}$
$\cos x = \frac{1}{\sec x}$	$\sec x = \frac{1}{\cos x}$
$\tan x = \frac{1}{\cot x}$	$\cot x = \frac{1}{\tan x}$
Quotient Identities	
$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$
Pythagorean Identities	
$\sin^2 x + \cos^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
	$1 + \cot^2 x = \csc^2 x$
Odd Identities	Even Identities
$\sin(-x) = -\sin x$ $\csc(-x) = -\csc x$ $\tan(-x) = -\tan x$ $\cot(-x) = -\cot x$	$\cos(-x) = \cos x$ $\sec(-x) = \sec x$

The Reduction Identities	
$\sin(x + \pi) = -\sin x$	$\sin\left(x + \frac{\pi}{2}\right) = \cos x$
$\cos(x + \pi) = -\cos x$	$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$
$\tan(x + \pi) = \tan x$	$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$

The Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

The Sum and Difference Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Half-Angle Formulas

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Product-to-Sum Formulas

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$