Angles

Measuring Angles in Degrees and in Radians

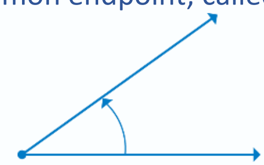
Definitions

Ray

Half of a line that has one point and continues indefinitely in one direction.

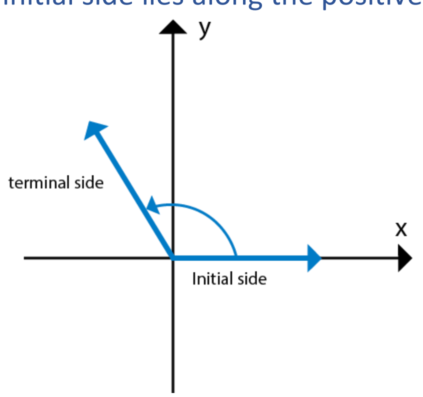
Angle

A geometric figure formed by two rays that have a common endpoint, called the vertex.



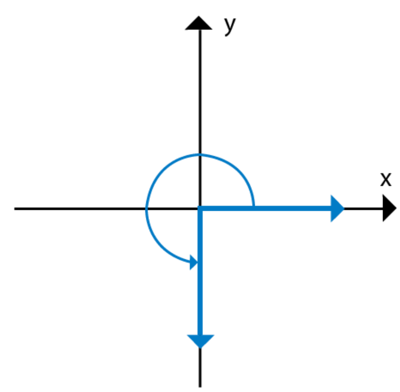
Angle in Standard Position

An angle whose vertex is at the origin of a rectangular coordinate system and whose initial side lies along the positive x-axis.



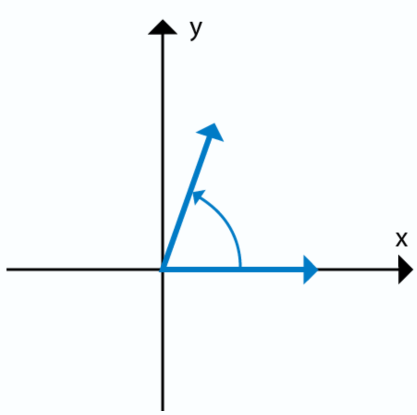
Quadrantal Angle

An angle whose terminal side lies on the x-axis or on the y-axis.



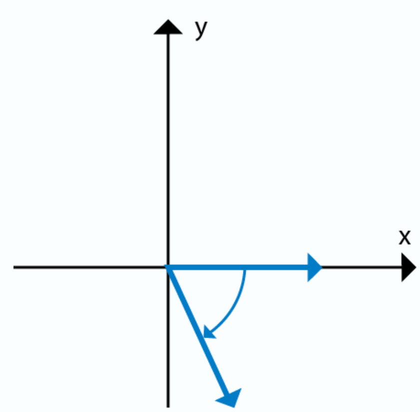
Positive Angle

An angle generated by counterclockwise rotation.



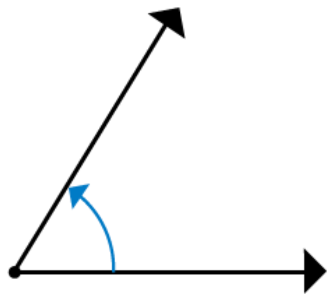
Negative Angle

An angle generated by clockwise rotation.



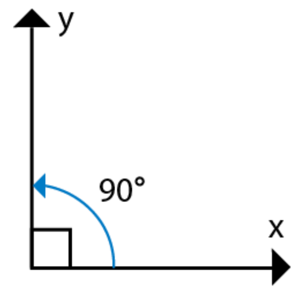
Acute Angle

An angle that measures less than 90°.



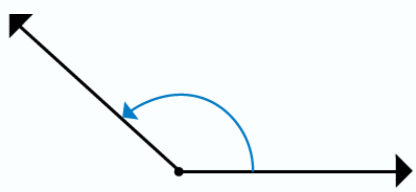
Right Angle

An angle that measures 90°.



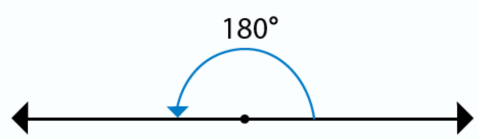
Obtuse Angle

An angle that measures more than 90°,
but less than 180°.



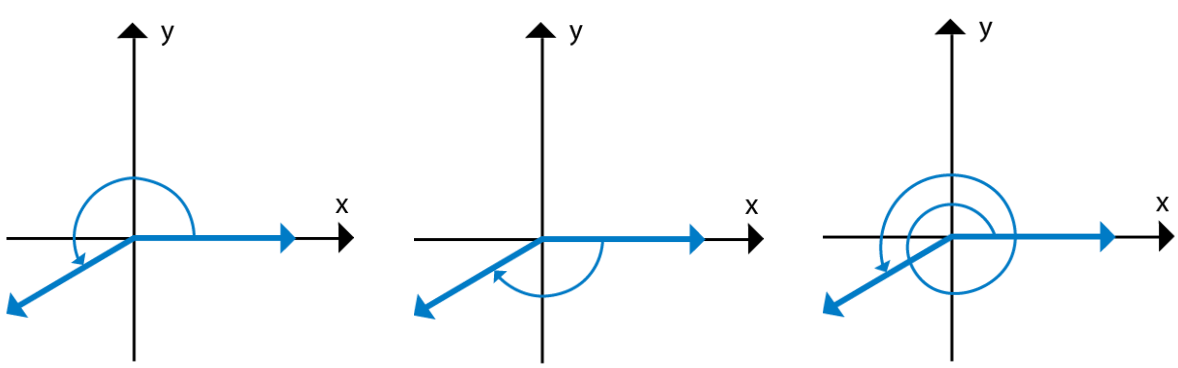
Straight Angle

An angle that measures 180°.



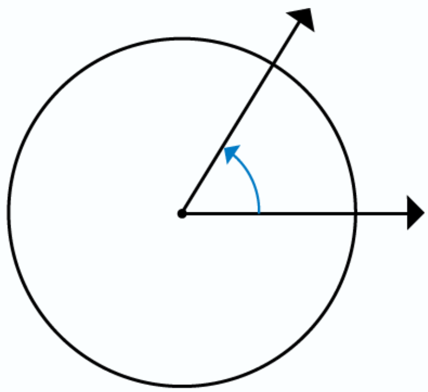
Coterminal Angles

Two or more angles with the same initial and terminal sides,
but possibly different rotations.



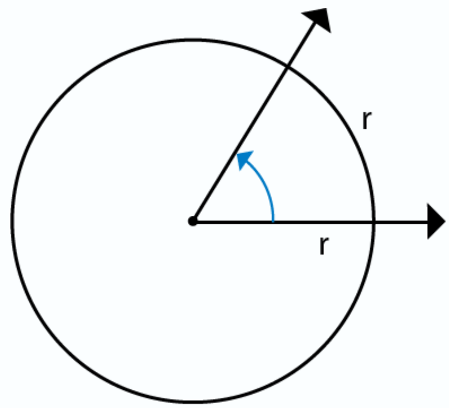
Central Angle

An angle whose vertex is at the center of a circle.



One Radian

The measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.



Degree

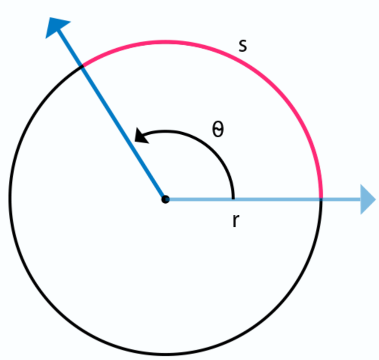
A degree $1°$ is a unit of measurement of angles and is $1/360$ of a complete rotation of a ray back into itself.

When writing the measure of angles in **degrees**, we **must** include the symbol °, for example $\theta = 35°$.

Radian Measure

The measure of the central angle θ, found by dividing the length of an arc s by the radius r of the circle.

$$\theta = \frac{s}{r}\ radians$$



When writing the measure of angles in radians, we may choose to include the word "radian" or you may choose not to include it, for example we may write $\theta = 3\ radians$ or we may write $\theta = 3$.

Example

Given the radius $r = 8\ inches$ and the arc length $s = 40\ inches$, find the radian measure.

$$\theta = \frac{s}{r} = \frac{40\ inches}{8\ inches} = 5\ radians$$

The Relationship Between Degrees and Radians

The circumference of a circle is $2\pi r$.
Use the formula for the radian measure $\theta = \frac{s}{r}$ to find the radian measure of one full rotation of a ray back into itself.

$$\theta = \frac{s}{r} = \frac{\text{the circumference}}{\text{r}} = \frac{2\pi r}{r} = 2\pi$$

So, one complete rotation in degrees is $360°$, but in radians is 2π radians.

$$360° = 2\pi$$

$$180° = \pi$$

If $180°$ is the same as π radians, then

$$\frac{180°}{\pi} = 1 \qquad and \qquad \frac{\pi}{180°} = 1$$

We can convert radians to degrees and degrees to radians using these two unit-fractions.

Converting Degrees to Radians	Converting Radians to Degrees
To convert degrees to radians, we multiply the degrees by $\frac{\pi}{180°}$.	To convert radians to degrees, we multiply the radians by $\frac{180°}{\pi}$.
Example 1 Convert $45°$ degrees to radians. $45° = 45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$	**Example 1** Convert $\frac{\pi}{2}$ radians to degrees. $\frac{\pi}{2} = \frac{\pi}{2} \cdot \frac{180°}{\pi} = 90°$
Example 2 Convert $-135°$ degrees to radians. $-135° = -135° \cdot \frac{\pi}{180°} = -\frac{3\pi}{4}$	**Example 2** Convert $\frac{\pi}{9}$ radians to degrees. $\frac{\pi}{9} = \frac{\pi}{9} \cdot \frac{180°}{\pi} = 20°$