The Greatest Common Factor

Definition of the Greatest Common Factor

The greatest common factor (GCF) of two or more of numbers is the largest number that will divide each of the given numbers exactly.

For example, 6 is the greatest common factor of 12 and 18, because 6 is the largest number that divides both 12 and 18.

Two Ways of Finding the Greatest Common Factor

The First Way

Find the greatest common factor (GCF) of 24 and 60.

Solution:

List all the factors of 24: 1, 2, 3, 4, 6, 8, **12**, 24
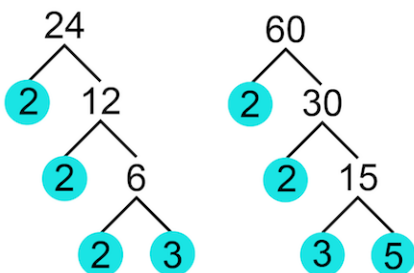List all the factors of 60: 1, 2, 3, 4, 5, 6, 10, **12**, 15, 20, 30, 60

The greatest number common to both lists is 12. So, $GCF = 12$.

The Second Way

Find the greatest common factor (GCF) of 24 and 60.

Solution:

Find the prime factorization of each number.

$$24 = 2^3 \cdot 3$$
$$60 = 2^2 \cdot 3 \cdot 5$$

Select each factor with the smallest exponent that is common to both factorizations, and form a product.

$$GCF = 2^2 \cdot 3$$

$$GCF = 12$$

The Least Common Multiple

Definition of a Multiple

A multiple of a number is the result of multiplying the number by an integer.
For example, the multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, …
Each number has infinitely many multiples.

Definition of the Least Common Multiple
(or the Least Common Denominator (LCD) when working with fractions)

The least common multiple (LCM) of two or more of numbers is the smallest number that is a multiple of these numbers.
For example, 12 is the least common multiple of 3 and 4, because 12 is the smallest number that is a multiple of both 3 and 4.

Two Ways of Finding the Least Common Multiple

The First Way

Find the least common multiple (LCM) of 8 and 12.
Solution:
List the first few multiples of 8:   8, 16, **24**, 32, 40, 48, 56, 64, 72, 80, …
List the first few multiples of 12:   12, **24**, 36, 48, 60, 72, 84, 96, …

As we can see, 8 and 12 have many of common multiples, like 24, 48, 72, and so on.
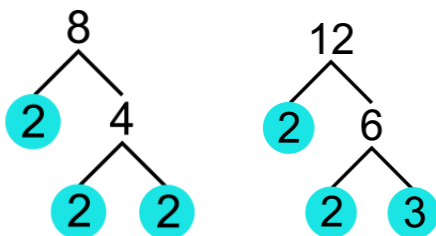But the least common multiple is 24. So, $LCM = 24.$

The Second Way

Find the least common multiple (LCM) of 8 and 12.
Solution:

Find the prime factorization of each number.

$$8 = 2^3$$
$$12 = 2^2 \cdot 3$$

Select each factor with the largest exponent that is either in one or the other or both factorizations, and form a product.

$$LCM = 2^3 \cdot 3$$

$$LCM = 24$$