Evaluating Algebraic Expressions

Definition of an Algebraic Expression

An algebraic expression is a collection of either numbers, variables, or both, connected with operations like addition, subtraction, multiplication, division, exponents, and/or radical symbols.

Examples:

$$2x + 5, \quad \sqrt{3y^2 + 7}, \quad 3a + 5b - 2c, \quad \frac{x + 5y}{3y^4}, \quad 3(x - 5) + 4x$$

When evaluating algebraic expressions, first replace all the letters with the given numbers, then use the order of operations to simplify the expressions.

The Order of Operations

Perform all operations inside parentheses, or any other grouping symbols. (Grouping symbols include parentheses, brackets, braces, absolute value, radicals, fraction bars.)
Evaluate all exponents, working from left to right.
Perform all multiplications and divisions, working from left to right.
Perform all additions and subtractions, working from left to right.

A common technique for remembering the order of operations is the abbreviation "**PEMDAS**", which is represented by the phrase "**Please Excuse My Dear Aunt Sally**" and it stands for "**Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction**".

Examples:

Examples:	Explanation:
Given $x = -3$ and $y = 5$, evaluate the algebraic expression. $$3x + 7y - 4$$ <u>Solution:</u> $$3x + 7y - 4$$ $$= 3(-3) + 7 \cdot 5 - 4$$ $$= -9 + 35 - 4$$ $$= 22$$	 Replace x and y with -3 and 5 respectively. Use parentheses when replacing a variable with a negative number. Perform the multiplications first. Add and subtract from left to right.
Given $x = 4$, $y = 8$, and $z = -5$, evaluate the algebraic expression. $$\frac{7x - y}{z}$$ <u>Solution:</u> $$\frac{7x - y}{z}$$ $$= \frac{7 \cdot 4 - 8}{-5}$$ $$= \frac{28 - 8}{-5}$$ $$= \frac{20}{-5}$$ $$= -4$$	 Replace all the variables with numbers. Perform the multiplication first. Subtract $28 - 8$. Divide 20 by -5.

Given $a = 6$, $b = 5$, and $c = -2$, evaluate the algebraic expression.

$$\frac{3a - 2b}{5c + a}$$

Solution:

Replace all the variables with numbers.

$$\frac{3 \cdot 6 - 2 \cdot 5}{5 \cdot (-2) + 6}$$

Perform all the multiplications first.

$$= \frac{18 - 10}{-10 + 6}$$

Subtract $18 - 10$ and add $-10 + 6$.

$$= \frac{8}{-4}$$

Divide 8 by -4.

$$= -2$$

Given $x = -3, y = 2$, and $z = -1$, evaluate the algebraic expression.

$$(x + 3y - 7z)(3x^2 - 20)$$

Solution:

$$(x + 3y - 7z)(3x^2 - 20)$$

Replace all the variables with numbers. Use parentheses when replacing a variable with a negative number.

$$= \left(-3 + 3 \cdot 2 - 7(-1)\right)(3(-3)^2 - 20)$$

Perform all the multiplications in the first parenthesis, and the exponent in the second parenthesis.

$$= (-3 + 6 + 7)(3 \cdot 9 - 20)$$

In the first parenthesis, add and subtract from left to right. In the second parenthesis, perform the multiplication.

$$= (10)(27 - 20)$$

$$= (10)(7)$$

Subtract $27 - 20$.

$$= 70$$

Multiply 10 and 7.