Exponential Expressions

For any natural number n and real number a,

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n}.$$

We call a the **base**, n the **exponent**, and a^n an **exponential expression**.

The exponent tells us how many times we multiply the given number.

Examples:

$$4^2 = 4 \cdot 4 = 16$$

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$5^1 = 5$$

Interesting Fact

The word “exponent” was introduced in 1544,
by the German monk and mathematician, Michael Stifel.

The Order of Operations

Perform all operations inside parentheses, or any other grouping symbols.
Evaluate all exponents, working from left to right.
Perform all multiplications and divisions, working from left to right.
Perform all additions and subtractions, working from left to right.

A common technique for remembering the order of operations is the abbreviation "**PEMDAS**", which is represented by the phrase "**Please Excuse My Dear Aunt Sally**" and it stands for "**Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction**".

Examples:

In evaluating expressions, you can perform several operations in the same step if they don't interfere with each other.

Evaluate. $(7 + 20) \div 3 - 4 \cdot 2$ $= 27 \div 3 - 8$ $= 9 - 8$ $= 1$	Perform addition inside the parentheses, and in the same step, multiply $4 \cdot 2$ to get 8. Now divide $27 \div 3$ to get 9. At last, subtract $9 - 8$ to get 1.
Evaluate. $5 + 4(7 - 3) + 10$ $= 5 + 4 \cdot 4 + 10$ $= 5 + 16 + 10$ $= 31$	Perform subtraction inside the parentheses. **(Do not add 5 and 4, because 4 must be multiplied by the result from the inside the parentheses).** Now multiply $4 \cdot 4$ to get 16. At last, add all three numbers together.

Evaluate. $[20-(8+2)]\div 5+4$ $=[20-10]\div 5+4$ $=10\div 5+4$ $=2+4$ $=6$	Add 8 and 2 inside parentheses. (They are the innermost grouping symbols). Subtract 10 from 20 inside the brackets. Divide 10 by 5. At last, add 2 and 4 together.
Evaluate. $\frac{8-2\cdot 3+2^3}{5(4-3)}$ $=\frac{8-6+8}{5\cdot 1}$ $=\frac{10}{5}$ $=2$	Simplify the top and the bottom separately. On the top, although the **exponent must be done** first, you can also **multiply 2 and 3** in the same step. These two operations do not interfere with each other in this exercise. On the bottom, subtract 3 from 4 inside the parentheses. On the top, perform $8-6+8$. On the bottom, multiply 5 and 1. Divide 10 by 5.
Evaluate. $7[2^2+(7-2)\cdot 2]-30\cdot 2$ $=7[4+5\cdot 2]-60$ $=7[4+10]-60$ $=7\cdot 14-60$ $=98-60$ $=38$	Inside the brackets, although the operation inside the **parentheses must be done first**, you can also **raise 2 to the second power**. And you can **multiply 30 and 2** all in the same step. These three operations do not interfere with each other in this exercise. Next, multiply 5 and 2. Add 4 and 10. Multiply 7 and 14. Subtract 98 and 60.

Evaluate. $\frac{5^2 + 1^5 - 2^3}{10 \div 5 \cdot 1 \cdot 4 \div 4}$ $= \frac{25 + 1 - 8}{2 \cdot 1 \cdot 4 \div 4}$ $= \frac{18}{2 \cdot 4 \div 4}$ $= \frac{18}{8 \div 4}$ $= \frac{18}{2}$ $= 9$	Simplify the top and the bottom separately. On the top, do all three exponents. On the bottom, perform multiplication and division as they appear from left to right, that is, first you will divide 10 by 5. On the top, perform $25 + 1 - 8$. On the bottom, multiply 2 and 1. On the bottom, multiply 2 and 4. On the bottom, divide 8 by 4. At last, divide 18 by 2.
Evaluate. $8 \div 2 \cdot 6 + 15$ $= 4 \cdot 6 + 15$ $= 24 + 15$ $= 39$	Divide 8 by 2. Now multiply 4 and 6. At last, add 24 and 15.