A Smart Inscription on Your Birthday Cake

A mathematician wrote on his son's birthday cake:

Happy

$$\frac{3^{\frac{3}{2}} \ 3^{\frac{3}{4}}}{3^{\frac{1}{4}}}$$

Birthday!

How old is his son?

The expression

$$\frac{3^{\frac{3}{2}} \cdot 3^{\frac{3}{4}}}{3^{\frac{1}{4}}}$$

contains rational exponents (or fractions in the exponents). To simplify it, we need to use the following rules:

$$a^m \cdot a^n = a^{m+n} \quad and \quad \frac{a^m}{a^n} = a^{m-n}$$

In other words, we have to add the exponents on the top, then subtract the exponent on the bottom.

$$\frac{3^{\frac{3}{2}} \cdot 3^{\frac{3}{4}}}{3^{\frac{1}{4}}} = 3^{\frac{3}{2}+\frac{3}{4}-\frac{1}{4}} = 3^{\frac{3 \cdot 2}{2 \cdot 2}+\frac{3}{4}-\frac{1}{4}}$$

$$= 3^{\frac{6}{4}+\frac{3}{4}-\frac{1}{4}} = 3^{\frac{8}{4}} = 3^2 = 9$$

So, his son is 9 years old.

Try Your Own Inscription:

Age	Expression	Age	Expression
1	$\dfrac{5^{\frac{2}{3}} \cdot 5^{\frac{1}{4}}}{5^{\frac{11}{12}}}$	6	$\dfrac{16^{\frac{1}{2}} \cdot 9^{\frac{1}{2}}}{4^{\frac{1}{2}}}$
2	$\dfrac{2^{\frac{3}{5}} \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{10}}}$	7	$\dfrac{7^{\frac{1}{2}} \cdot 7^{\frac{3}{5}}}{7^{\frac{1}{10}}}$
3	$\dfrac{9^{\frac{4}{3}} \cdot 9^{\frac{2}{3}}}{9^{\frac{3}{2}}}$	8	$\dfrac{2^{\frac{3}{2}} \cdot 2^{\frac{7}{4}}}{2^{\frac{1}{4}}}$
4	$\dfrac{4^{\frac{3}{2}} \cdot 4^{\frac{7}{10}}}{4^{\frac{6}{5}}}$	9	$\dfrac{3^{\frac{5}{2}} \cdot 3^{\frac{3}{4}}}{3^{\frac{5}{4}}}$
5	$\dfrac{25^{\frac{1}{2}} \cdot 5^{\frac{2}{3}}}{5^{\frac{4}{6}}}$	10	$\dfrac{25^{\frac{1}{2}} \cdot 16^{\frac{1}{2}}}{4^{\frac{1}{2}}}$