## $1=2$

Here is a bizarre proof of $1=2$.
And here is how it goes:

| Let there be two positive unknown numbers $a$ and $b$. And let these numbers be equal. | $a=b$ |
| :---: | :---: |
| Now we will apply a sequence of operations to this equation: <br> First, we will multiply both sides by $a$. | $\begin{aligned} a \cdot a & =a \cdot b \\ a^{2} & =a b \end{aligned}$ |
| Next, we will subtract $b^{2}$ from both sides. | $a^{2}-b^{2}=a b-b^{2}$ |
| Factor the left side using the formula $a^{2}-b^{2}=(a+b)(a-b)$. <br> Factor the right side by factoring out $b$. | $(a+b)(a-b)=b(a-b)$ |
| Since both sides contain ( $a-b$ ), divide both sides of the equation by $(a-b)$. | $\frac{(a+b)(a-b)}{(a-b)}=\frac{b(a-b)}{(a-b)}$ |
| Because the initial problem states that $a=b$, we can replace $a$ with $b$. | $\begin{aligned} & a+b=b \\ & b+b=b \end{aligned}$ |
| Combine the like terms on the left side. | $2 b=b$ |
| Now, divide both sides by $b$, and cancel $b$. | $\frac{2 b}{b}=\frac{b}{b}$ |
| After canceling $b$, we get that $2=1$. | $2=1$ |

Now, we all know that 2 is not equal to 1 . So, what happened in the process that gave us this bizarre answer?
It is the step where we divided both sides by $(a-b)$ :

$$
\frac{(a+b)(a-b)}{(a-b)}=\frac{b(a-b)}{(a-b)}
$$

In the beginning, we stated that $a=b$. So, if we subtract $a-b$, the answer will be equal to zero. And we know that division by zero is undefined (can't divide by zero). So, this step is invalid:

$$
\frac{(a+b)(a-b)}{(a-b)}=\frac{b(a-b)}{(a-b)}
$$

Therefore, we happily conclude that $1 \neq 2$.

