Week 9

Sections 3.6, 4.1, 4.2

HW8: 34, 38, 44, 52 (p. 309)
8, 12, 24, 30 (p. 336-337)
26, 28 (p. 342)

Review Exercises

Find all horizontal and vertical asymptotes (if any).

$$f(x) = \frac{x^3 - 8x}{x^4 - 16}$$

Solution

1. Find the vertical asymptotes (by setting the denominator equal to zero).

$$x^4 - 16 = 0$$

$$(x^2 + 4)(x^2 - 4) = 0$$

$$x^2 + 4 = 0 \quad or \quad x^2 - 4 = 0$$

$$x^2 = -4 \quad or \quad (x + 2)(x - 2) = 0$$

$$No\ solution \quad or \quad x + 2 = 0 \quad or \quad x - 2 = 0$$

$$x = -2 \quad or \quad x = 2$$

The vertical asymptotes are $x = -2$ and $x = 2$.

2. Find the horizontal asymptote.

The horizontal asymptote is $y = 0$
(because the exponent on the numerator is less than the exponent on the denominator).

Find all horizontal and vertical asymptotes (if any).

$$f(x) = \frac{(x+1)(2x-3)}{(x-2)(4x+7)}$$

<u>Solution</u>

1. <u>Find the vertical asymptotes (by setting the denominator equal to zero).</u>

$$(x-2)(4x+7) = 0$$
$$x - 2 = 0 \quad or \quad 4x + 7 = 0$$
$$x = 2 \quad or \quad 4x = -7$$
$$x = 2 \quad or \quad x = -\frac{7}{4}$$

The vertical asymptotes are:

$$x = 2 \quad and \quad x = -\frac{7}{4}$$

2. <u>Find the horizontal asymptote.</u>

The numerator: $(x+1)(2x-3) = 2x^2 - x - 3$

The highest exponent is 2.
The leading coefficient is **2**.

The denominator: $(x-2)(4x+7) = 4x^2 - x - 14$

The highest exponent is 2.
The leading coefficient is **4**.

Because both exponents of the numerator and the denominator are the same, we need to divide the coefficients to find the horizontal asymptote.

$$y = \frac{2}{4}$$

$$y = \frac{1}{2}$$

Find the intercepts and asymptotes, and then sketch a graph of the rational function and state the domain and range.

$$f(x) = \frac{4x - 4}{x + 2}$$

Solution

1. Find the x-intercepts.
   Replace y with 0.

$$\frac{4x - 4}{x + 2} = 0$$

A fraction is equal to zero only because the numerator is equal to zero:

$$4x - 4 = 0$$
$$4x = 4$$
$$x = 1$$

The x-intercept is 1

2. Find the y-intercept.
   Replace x with 0.

$$y = \frac{4 \cdot 0 - 4}{0 + 2}$$

$$y = -\frac{4}{2}$$

$$y = -2$$

The y-intercept is -2.

3. Find the vertical asymptotes (by setting the denominator equal to zero).

$$x + 2 = 0$$
$$x = -2$$

The vertical asymptote is: $x = -2$

4. Find the horizontal asymptote.

Because both exponents of the numerator and the denominator are the same, we need to divide the leading coefficients to find the horizontal asymptote.

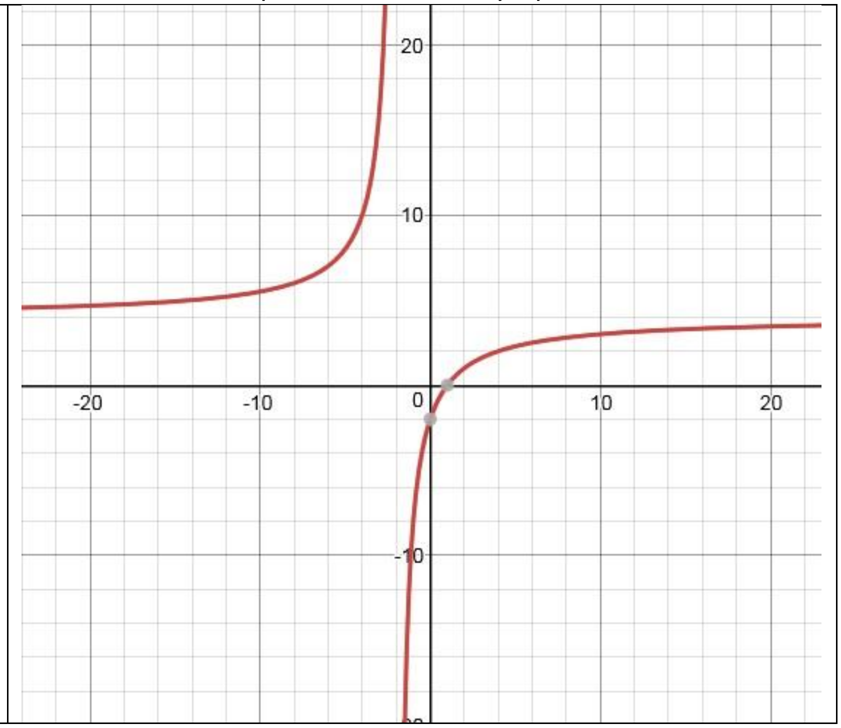
$$y = \frac{4}{1} = 4$$

The horizontal asymptote is: $y = 4$

Cont. on the next page…

5. <u>Get additional points</u> between the intercepts and vertical asymptotes.

x	y
-3	16
-1	-8
2	1
3	1.6



Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, 4) \cup (4, \infty)$

Find the intercepts and asymptotes, and then sketch a graph of the rational function and state the domain and range.

$$f(x) = \frac{4x - 8}{(x-4)(x+1)}$$

Solution

1. Find the x-intercepts.
   Replace y with 0.

$$\frac{4x - 8}{(x-4)(x+1)} = 0$$

A fraction is equal to zero only because the numerator is equal to zero:

$$4x - 8 = 0$$
$$4x = 8$$
$$x = 2$$

The x-intercept is 2

2. Find the y-intercept.
   Replace x with 0.

$$y = \frac{4 \cdot 0 - 8}{(0-4)(0+1)}$$

$$y = \frac{-8}{-4}$$

$$y = 2$$

The y-intercept is 2.

3. Find the vertical asymptotes (by setting the denominator equal to zero).

$$(x-4)(x+1) = 0$$
$$x - 4 = 0 \quad or \quad x + 1 = 0$$
$$x = 4 \quad or \quad x = -1$$

The vertical asymptotes are: $x = 4$ and $x = -1$

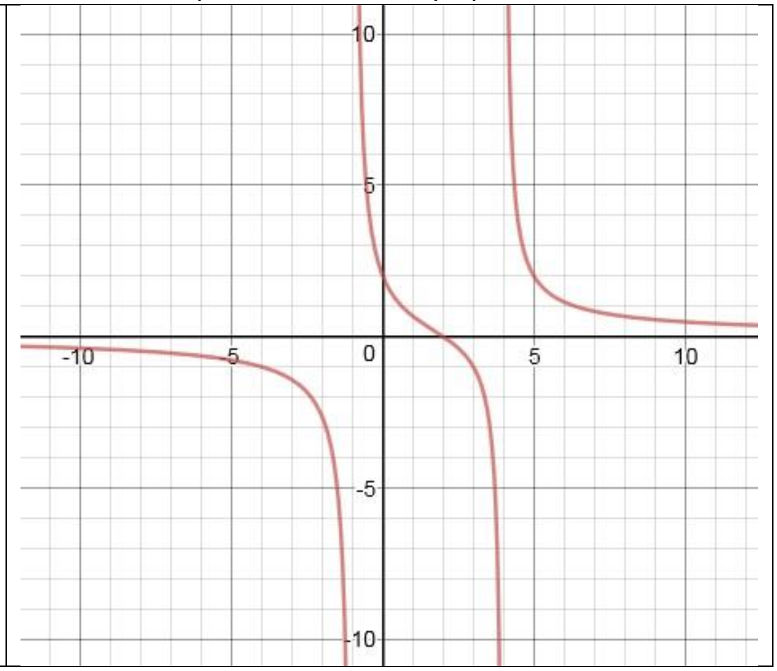
4. Find the horizontal asymptote.

The horizontal asymptote is $y = 0$
(because the exponent on the numerator is less than the exponent on the denominator).

Cont. on the next page…

5. <u>Get additional points</u> between the intercepts and vertical asymptotes.

x	y
-3	-1.4
-2	-2.7
0	2
3	-1
5	2



Domain: $(-\infty,-1) \cup (-1,4) \cup (4,\infty)$
Range: $(-\infty,\infty)$

Use a calculator to evaluate the function at the indicated values. Round your answers to three decimals (that is three digits after the decimal point).

$$f(x) = 2^{x-3}$$

$$f\left(\frac{1}{2}\right), \qquad f(2.5), \qquad f(-1), \qquad f\left(\frac{1}{4}\right)$$

<u>Solution</u>

$$f\left(\frac{1}{2}\right) = 2^{\frac{1}{2}-3} \approx 0.17677669 \ldots \approx 0.177$$

$$f(2.5) = 2^{2.5-3} \approx 0.7071067 \ldots \approx 0.707$$

$$f(-1) = 2^{-1-3} = 0.0625 \approx 0.063$$

$$f\left(\frac{1}{4}\right) = 2^{\frac{1}{4}-3} \approx 0.14865 \ldots \approx 0.149$$

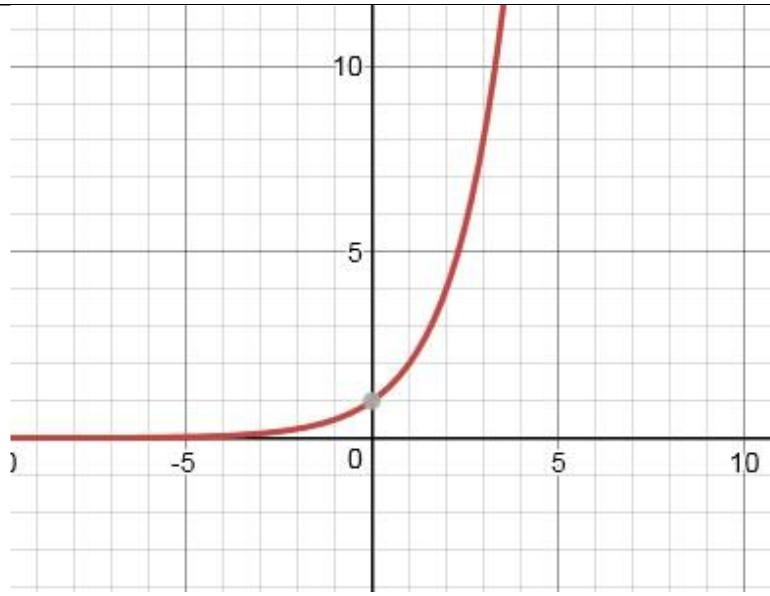
Note:
On some of the calculators you may have to type the exponent in parentheses. For example: 2^(1/2-3)

Sketch the graph of the function by making a table of values. Use a calculator if necessary.

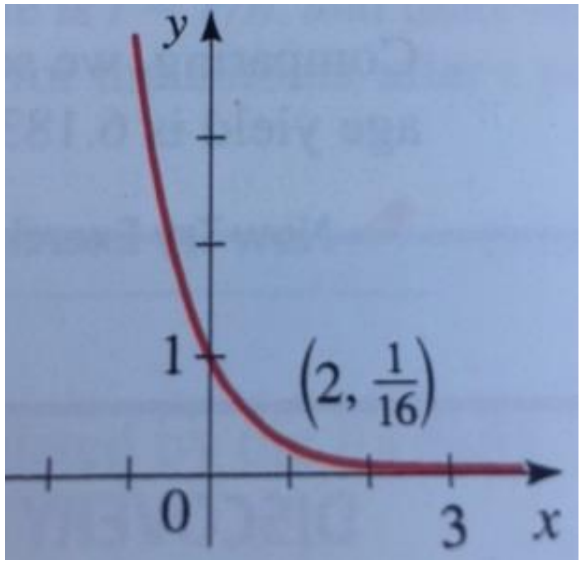
$$f(x) = 2^x$$

Solution

x	y
-4	$\frac{1}{16}$
-2	$\frac{1}{4}$
0	1
2	4
4	16



Find the exponential function $f(x) = a^x$ whose graph is given.



Solution

The graph has the point: $\left(2, \frac{1}{16}\right)$

$$x = 2, \qquad y = \frac{1}{16}$$

Use these numbers to find a.

$$a^x = y$$

$$a^2 = \frac{1}{16}$$

$$a = \sqrt{\frac{1}{16}}$$

$$a = \frac{1}{4}$$

So, the exponential function is:

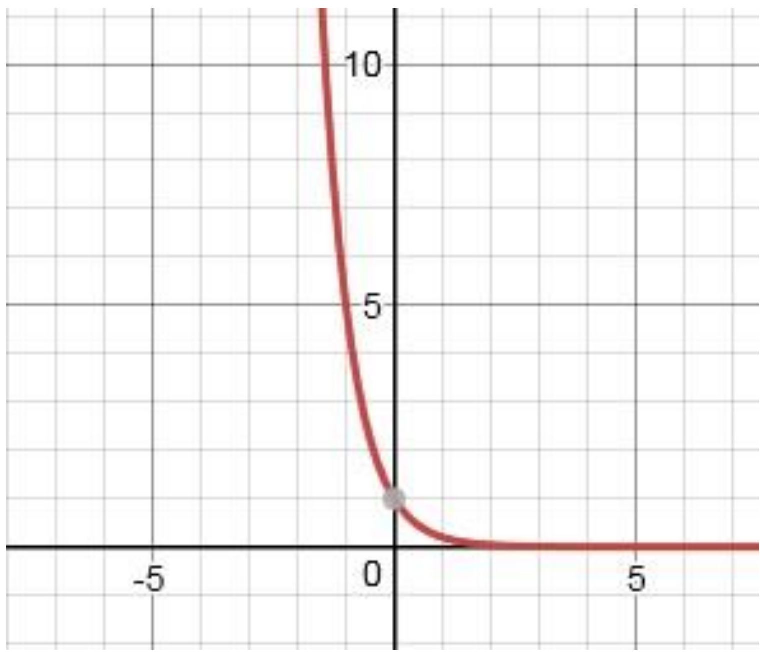
$$f(x) = \left(\frac{1}{4}\right)^x$$

Graph the function not by plotting the points, but by starting from the basic exponential function. State the domain, range and asymptote.

$$f(x) = 5^{-x}$$

Solution

The graph of $f(x) = 5^{x}$is obtained by reflecting the graph of $y = 5^{x}$ about the y-axis.



$$Domain: (-\infty, \infty)$$

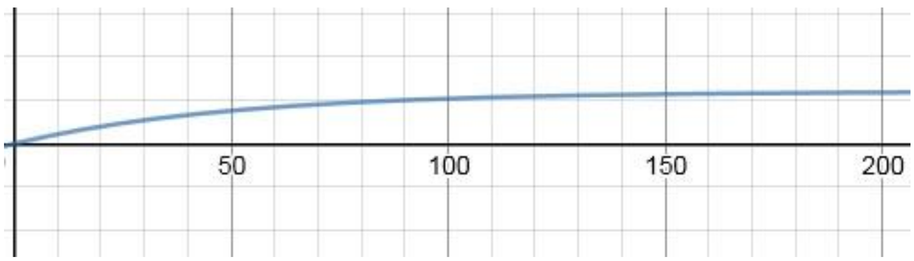
$$Range: (0, \infty)$$

$$Asymptote: y = 0$$

The amount of salt in a barrel at time t is given by $Q(t) = 12(1 - e^{-0.02t})$ where t is measured in minutes and $Q(t)$ is measured in pounds.

a. How much salt is in the barrel after 5 minutes?
b. How much salt is in the barrel after 10 minutes?
c. Draw a graph of the function $Q(t)$?
d. Use the graph in part (c) to determine the value that the amount of salt in the barrel approaches as t becomes large. Is this what you would expect?

Solution

a. $Q(5) = 12(1 - e^{-0.02 \cdot 5}) \approx 1.142$
b. $Q(10) = 12(1 - e^{-0.02 \cdot 10}) \approx 2.175$
c. 

d. The amount of salt of salt approaches 12 lb.

The population of a certain species of bird behaves according to the logistic growth model

$$n(t) = \frac{4000}{0.5 + 26.4e^{-0.044t}}$$

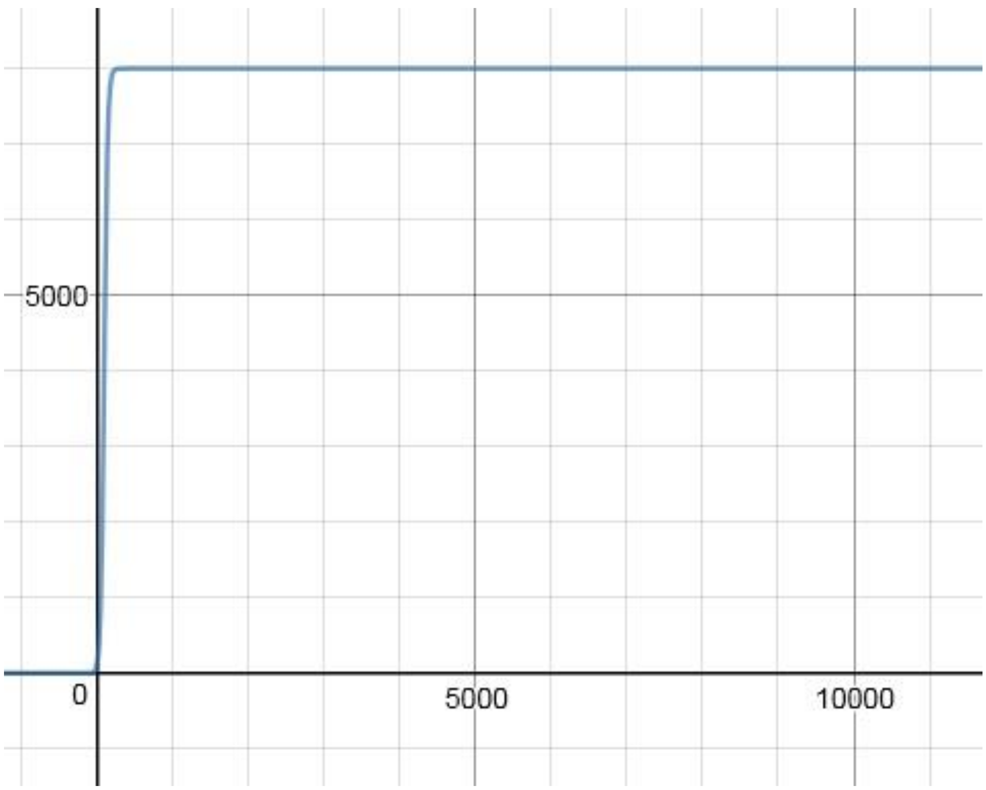
where t is measured in years.

a. Find the initial bird population.
b. Draw a graph of the function $n(t)$.
c. What size does the population approaches time goes on?

<u>Solution</u>

a. $n(0) = \frac{4000}{0.5+26.4e^{-0.044\cdot 0}} = 148.7$

b.



c. From the graph, the population approaches 8000 birds as t increases.