Week 8

Sections 3.2, 3.3, 3.4

HW8: 18, 20, 32, 52 (p. 266-267)
6, 28, 30 (p. 273-274)
6, 16, 18 (p. 283)

Review Exercises

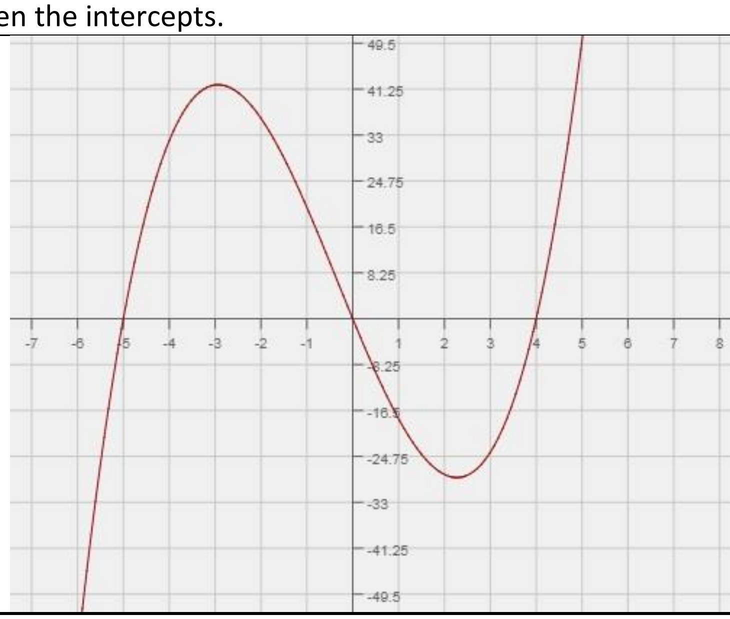
Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$$f(x) = x(x-4)(x+5)$$

Solution

1. Find the function's end behavior.
   If we would multiply all the factors and remove the parentheses, then the term with the largest exponent would be x^3.
   Because the coefficient of x^3 is positive, and the exponent is odd, the **graph will fall to the left and will rise to the right**.
2. Find the x-intercepts.
   Replace y with 0.
   $$x(x-4)(x+5) = 0$$
   $$x = 0 \quad or \quad x-4=0 \quad or \quad x+5=0$$
   $$x = 0 \quad or \quad x=4 \quad or \quad x=-5$$
3. Find the y-intercept.
   Replace x with 0.
   $$y = 0(0-4)(0+5)$$
   $$y = 0$$
4. Get additional points between the intercepts.

x	y
-6	-60
-3	42
-1	20
1	-18
2	-28
5	50



Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$$f(x) = (x+1)(x-4)(2x+5)$$

Solution

1. Find the function's end behavior.
   If we would multiply all the factors and remove the parentheses, then the term with the largest exponent would be $2x^3$.
   Because the coefficient 2 of x^3 is positive, and the exponent is odd, the **graph will fall to the left and will rise to the right**.

2. Find the x-intercepts.
   Replace y with 0.

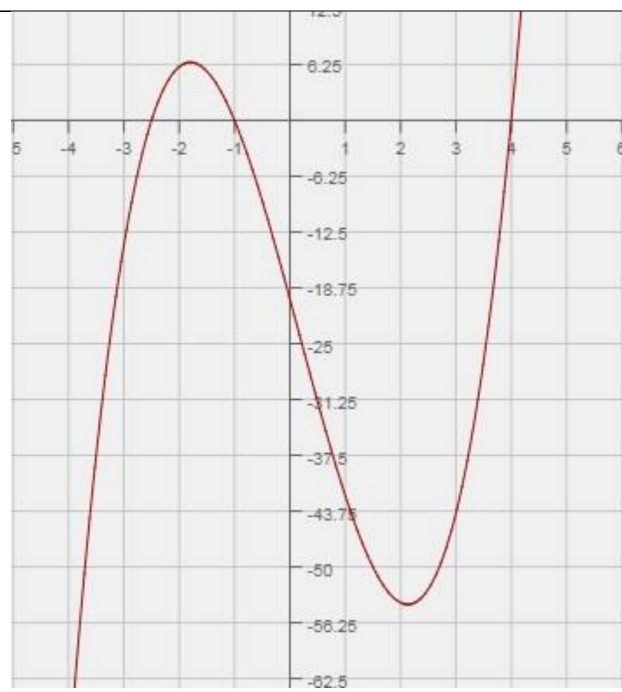
$$(x+1)(x-4)(2x+5) = 0$$
$$x+1=0 \quad or \quad x-4=0 \quad or \quad 2x+5=0$$
$$x=-1 \quad or \quad x=4 \quad or \quad 2x=-5$$
$$x=-1 \quad or \quad x=4 \quad or \quad x=-\frac{5}{2}$$

3. Find the y-intercept.
   Replace x with 0.

$$y = (0+1)(0-4)(2\cdot 0+5)$$
$$y = (1)(-4)(5)$$
$$y = -20$$

4. Get additional points between the intercepts.

x	y
-3	-14
-2	6
2	-54
5	90



Factor the polynomial and use the factored form to find the zeros. Then sketch the graph.

$$f(x) = x^3 - x^2 - 12x$$

Solution

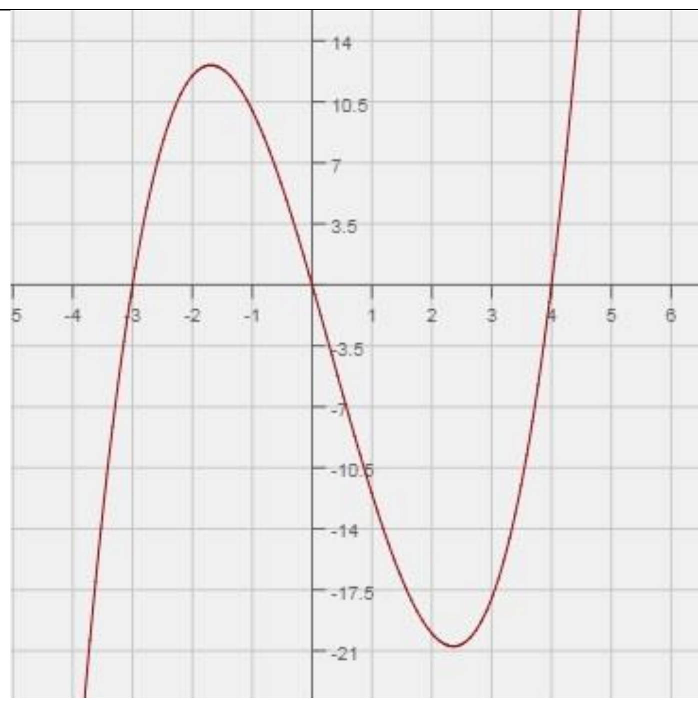
$$f(x) = x(x^2 - x - 12)$$
$$f(x) = x(x - 4)(x + 3)$$

1. Find the function's end behavior.
   Because the coefficient of x^3 is positive, and the exponent is odd, the **graph will fall to the left and will rise to the right**.

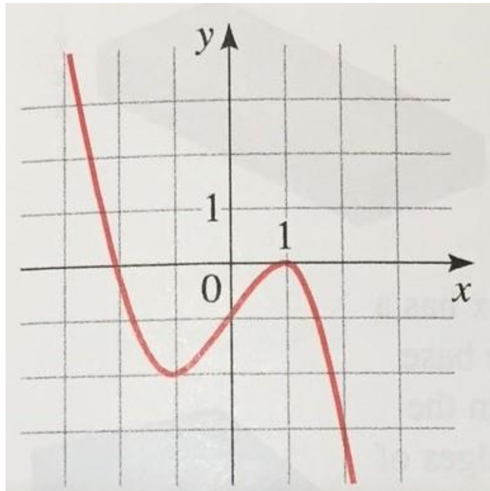
2. Find the x-intercepts.
   Replace y with 0.
   $$x(x - 4)(x + 3) = 0$$
   $$x = 0 \quad or \quad x - 4 = 0 \quad or \quad x + 3 = 0$$
   $$x = 0 \quad or \quad x = 4 \quad or \quad x = -3$$
3. Find the y-intercept.
   Replace x with 0.
   $$y = 0(0 - 4)(0 + 3)$$
   $$y = 0$$

4. Get additional points between the intercepts.

x	y
-4	-32
-2	12
-1	10
1	-12
2	-20
5	40



The graph of a polynomial function is given. From the graph, find a) the x- and y-intercepts, and b) the coordinates of all local extrema.



Solution

a) $x - intercepts: -2, 1; \quad y - intercept: -1$
b) Local Maximum: $(1, 0)$; Local Minimum: $(-1, -2)$

Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

$$P(x) = 8x^3 + 4x^2 - 10x + 9$$
$$D(x) = 2x - 3$$

Solution

$$\begin{array}{r} 4x^2 + 8x + 7 \\ 2x - 3\,\overline{)8x^3 + 4x^2 - 10x + 9} \\ \underline{-(8x^3 - 12x^2)} \\ 16x^2 - 10x \\ \underline{-(16x^2 - 24x)} \\ 14x + 9 \\ \underline{-(14x - 21)} \\ 30 \end{array}$$

$$\frac{P(x)}{D(x)} = 4x^2 + 8x + 7 + \frac{30}{2x - 3}$$

Find the quotient and the remainder using synthetic division.

$$\frac{5x^2 - 4}{x - 3}$$

Solution

$$\begin{array}{r|rrr} 3 & 5 & 0 & -4 \\ & & 15 & 45 \\ \hline & 5 & 15 & 41 \end{array}$$

$$5x + 15 \quad R41$$

Find the quotient and the remainder using synthetic division.

$$\frac{x^3 + 2x^2 + 2x + 1}{x + 2}$$

Solution

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 2 & 1 \\ & & -2 & 0 & -4 \\ \hline & 1 & 0 & 2 & -3 \end{array}$$

$$x^2 + 2 \quad R - 3$$

List all possible rational zeros given by the Rational Zeros Theorem.

$$R(x) = x^3 + 5x - 6$$

Solution

$$p = \pm 1, \pm 2, \pm 3, \pm 6$$

$$q = \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$

All the real zeros of the given polynomial are integers. Find the zeros, and write the polynomial in factored form.

$$P(x) = x^3 + 2x^2 - 13x + 10$$

Solution

$$p = \pm 1, \pm 2, \pm 5, \pm 10$$

$$q = \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10$$

Use synthetic division to test the possible rational roots and find an actual root.

Try number 1.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -13 & 10 \\ & & 1 & 3 & -10 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

The remainder is zero, so 1 is a rational root, and $x - 1$ is a factor.

Factor completely and solve the equation to find the zeros.

$$(x-1)(x^2 + 3x - 10) = 0$$

$$(x-1)(x+5)(x-2) = 0$$

$$x - 1 = 0 \quad or \quad x + 5 = 0 \quad or \quad x - 2 = 0$$

$$x = 1 \quad or \quad x = -5 \quad or \quad x = 2$$

All the real zeros of the given polynomial are integers. Find the zeros, and write the polynomial in factored form.

$$P(x) = x^3 - 19x - 30$$

Solution

$$p = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$$

$$q = \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$$

Use synthetic division to test the possible rational roots and find an actual root.

Try number -2.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

The remainder is zero, so -2 is a rational root, and $x + 2$ is a factor.

Factor completely and solve the equation to find the zeros.

$$(x + 2)(x^2 - 2x - 15) = 0$$

$$(x + 2)(x + 3)(x - 5) = 0$$

$$x + 2 = 0 \quad or \quad x + 3 = 0 \quad or \quad x - 5 = 0$$

$$x = -2 \quad or \quad x = -3 \quad or \quad x = 5$$