# Week 7

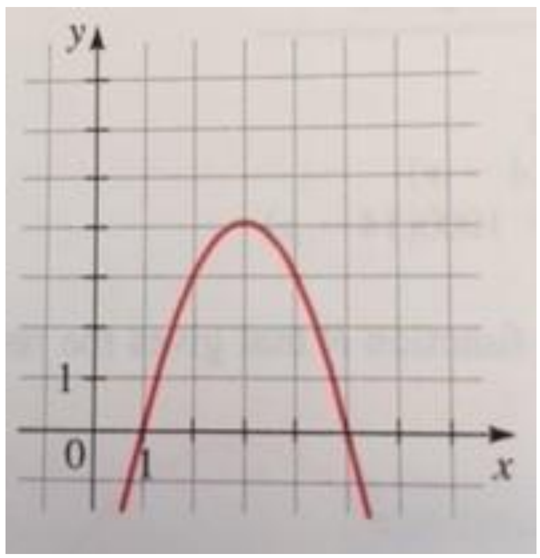
## Sections 3.1

HW7: 6, 8, 10, 18, 36, 44, 54, 58 (p. 252-253)

### Review Exercises

The graph of a quadratic function is given.

a) Find the coordinates of the vertex and the x- and y-intercepts.
b) Find the maximum or minimum value of $f$.
c) Find the domain and range of $f$.




<u>Solution</u>

a) Vertex: $(3, 4)$
   x-intercepts: 1 and 5
   y-intercept: -5 (not seen on the graph, but assumed)

b) Max: 4
   No Min.

c) Domain: $(-\infty, \infty)$
   Range: $(-\infty, 4]$

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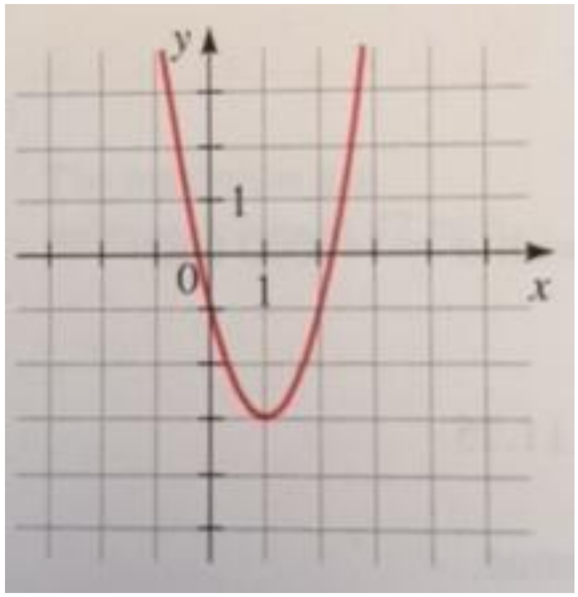


<u>Solution</u>

a) Vertex: $(1, -3)$
x-intercepts: approximately $-0.2$ and $2.2$
y-intercept: $-1$

b) No Max.
No Min: $-3$

c) Domain: $(-\infty, \infty)$
Range: $[3, \infty)$

A quadratic function is given.

$$f(x) = x^2 - 2x + 3$$

a) Express $f$ in standard form.
b) Find the vertex and x- and y-intercepts of $f$.
c) Sketch a graph of $f$.
d) Find the domain and range of $f$.

Solution

a) $f(x) = x^2 - 2x + 3$

$$f(x) = \left(x^2 - 2x + \left(\frac{2}{2}\right)^2\right) + 3 - \left(\frac{2}{2}\right)^2$$

$$f(x) = \left(x^2 - 2x + \left(\frac{2}{2}\right)^2\right) + 3 - \left(\frac{2}{2}\right)^2$$

$$f(x) = (x^2 - 2x + 1) + 3 - 1$$

$$f(x) = (x - 1)^2 + 2$$

b) Vertex: $(1, 2)$
x-intercepts:
(replace $y$ with 0 and solve for $x$)
$(x - 1)^2 + 2 = 0$
$(x - 1)^2 = -2$
No solution, because $(x - 2)^2$ cannot be equal to a negative number.
Therefore, there are no x-intercepts.

y-intercept:
(replace $x$ with 0 and solve for $y$)
$y = (0 - 1)^2 + 2$
$y = 1 + 2$
$y = 3$

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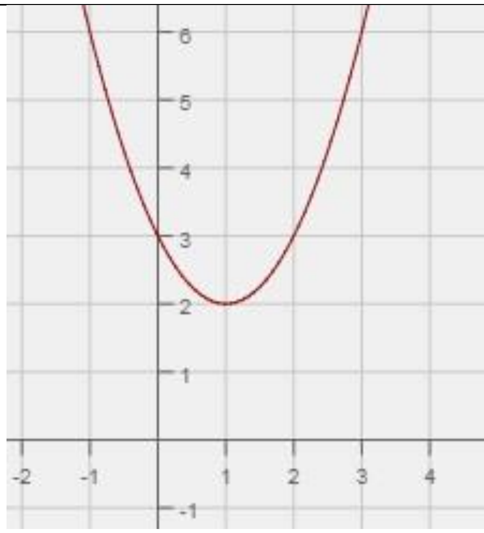
c) 


d) Domain: $(-\infty, \infty)$
Range: $[2, \infty)$

A quadratic function is given.

$$f(x) = -x^2 + 6x + 4$$

e) Express $f$ in standard form.
f) Find the vertex and x- and y-intercepts of $f$.
g) Sketch a graph of $f$.
h) Find the domain and range of $f$.

<u>Solution</u>

d) $f(x) = -x^2 + 6x + 4$

$$f(x) = -\left(x^2 - 6x + \left(\frac{6}{2}\right)^2\right) + 4 + \left(\frac{6}{2}\right)^2$$

$$f(x) = -\left(x^2 - 6x + \left(\frac{6}{2}\right)^2\right) + 4 + \left(\frac{6}{2}\right)^2$$

$$f(x) = -(x^2 - 6x + 9) + 4 + 9$$

$$f(x) = -(x - 3)^2 + 13$$

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e) Vertex: $(3, 13)$

x-intercepts:

(replace $y$ with 0 and solve for $x$)

$-(x-3)^2 + 13 = 0$

$-(x-3)^2 = -13$

$(x-3)^2 = 13$

$x - 3 = \pm\sqrt{13}$

$x = 3 \pm \sqrt{13}$

y-intercept:

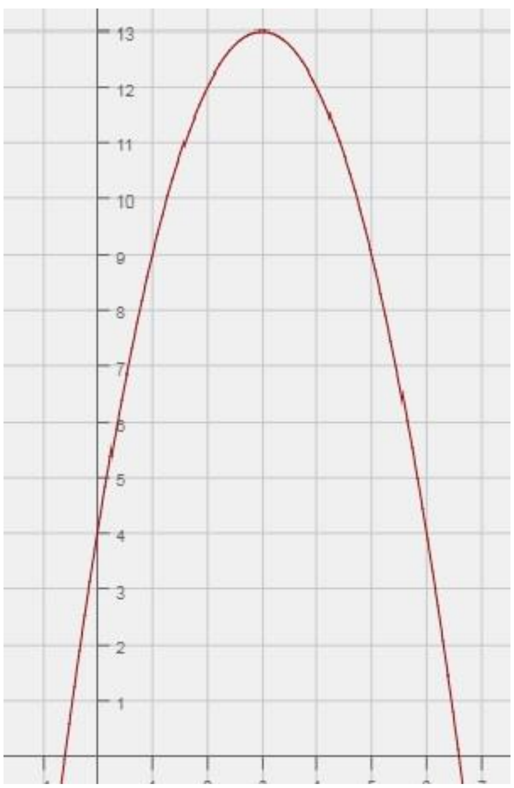
(replace $x$ with 0 and solve for $y$)

$y = -(x-3)^2 + 13$

$y = -(0-3)^2 + 13$

$y = -9 + 13$

$y = 4$

f) 


e) Domain: $(-\infty, \infty)$

Range: $(-\infty, 13]$

Find the maximum or minimum value of the function.

$$f(x) = -1 + 4x + 2x^2$$

Solution

Rearrange the terms.

$$f(x) = 2x^2 + 4x - 1$$

The leading coefficient 2 is positive, therefore, we will have a minimum value.

$$x = -\frac{b}{2a} = -\frac{4}{2 \cdot 2} = -1$$

$$f(-1) = 2(-1)^2 + 4(-1) - 1$$

$$f(-1) = 2 - 4 - 1$$

$$f(-1) = -3$$

$$Min = -3$$

Find the maximum or minimum value of the function.

$$f(x) = 3x(x - 2) + 5$$

Solution

Remove the parentheses.

$$f(x) = 3x^2 - 6x + 5$$

The leading coefficient 3 is positive, therefore, we will have a minimum value.

$$x = -\frac{b}{2a} = -\frac{-6}{2 \cdot 3} = 1$$

$$f(1) = 3 \cdot 1^2 - 6 \cdot 1 + 5$$

$$f(1) = 3 - 6 + 5$$

$$f(1) = 2$$

$$Min = 2$$

A soft-drink vendor at a popular beach analyzes his sales records and finds that if he sells $x$ cans of soda pop in one day, his profit (in dollars) is given by

$$P(x) = -0.002x^2 + 4x - 1800$$

What is his maximum profit per day, and how many cans must he sell for maximum profit?

<u>Solution</u>

$$x = -\frac{4}{2(-0.002)} = -\frac{4}{-0.004} = 1000$$

$$P(1000) = -0.002(1000)^2 + 4(1000) - 1800$$

$$P(x) = \$200$$

At a certain vineyard it is found that each grape vine produces about 12 lb of grapes in a season when about 750 vines are planted per acre. For each additional vine that is planted, the production of each vine decreases by about 1 percent. So the number of pounds of grapes produced per acre is modeled by $A(n) = (750 + n)(12 - 0.01n)$ where $n$ is the number of additional vines planted. Find the number of vines that should be planted to maximize grape production.

<u>Solution</u>

Remove the parentheses.

$$A(n) = (750 + n)(12 - 0.01n)$$
$$A(n) = 9000 - 7.5n + 12n - 0.01n^2$$
$$A(n) = -0.01n^2 + 4.5n + 9000$$

$$n = -\frac{b}{2a} = -\frac{4.5}{2(-0.01)} = 225$$

$$750 + 225 = 975\ vines$$