Week 6

Sections 2.7

HW6: 8, 12, 20, 28, 30, 48, 50, 60 (p. 216-217)

Review Exercises

Given $f(x) = x + 5, \quad g(x) = \sqrt{x}$

Find $f + g, f - g, fg$ and ${}^{f}/_{g}$ and their domain.

<u>Solution</u>

$f + g = x + 5 + \sqrt{x}$ $Domain: [0, \infty)$	$f - g = x + 5 - \sqrt{x}$ $Domain: [0, \infty)$	$fg = (x + 5)\sqrt{x}$ $= x\sqrt{x} + 5\sqrt{x}$ $Domain: [0, \infty)$	$\frac{f}{g} = \frac{x + 5}{\sqrt{x}}$ $Domain: (0, \infty)$

Given $f(x) = x^2 + 3x, \quad g(x) = 4x^2 - 1$

Find $f + g, f - g, fg$ and ${}^{f}/_{g}$ and their domain.

<u>Solution</u>

$f + g = (x^2 + 3x) + (4x^2 - 1)$ $= 5x^2 + 3x - 1$ $Domain: (-\infty, \infty)$	$f - g = (x^2 + 3x) - (4x^2 - 1)$ $= x^2 + 3x - 4x^2 + 1$ $= -3x^2 + 3x + 1$ $Domain: (-\infty, \infty)$
$fg = (x^2 + 3x)(4x^2 - 1)$ $= 4x^4 - x^2 + 12x^3 - 3x$ $Domain: (-\infty, \infty)$	$\frac{f}{g} = \frac{x^2 + 3x}{4x^2 - 1}$ $4x^2 - 1 \neq 0$ $4x^2 \neq 1$ $x^2 \neq \frac{1}{4}$ $x \neq \pm\frac{1}{2}$ $Domain: \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Find the domain of the function

$$f(x) = \frac{\sqrt{x+6}}{x-5}$$

Solution

$$x + 6 \geq 0 \quad and \quad x - 5 \neq 0$$
$$x \geq -6 \quad and \quad x \neq 5$$

$$Domain: [-6, 5) \cup (5, \infty)$$

Given

$$f(x) = 4x - 5 \quad and \quad g(x) = 2 - x^2$$

Evaluate the expressions.

a) $f(f(2))$
b) $g(g(2))$

Solution

a) $f\big(f(2)\big) = f(4 \cdot 2 - 5) = f(3) = 4 \cdot 3 - 5 = 7$
b) $g\big(g(2)\big) = g(2 - 2^2) = g(-2) = 2 - (-2)^2 = 2 - 4 = -2$

Given

$$f(x) = 4x - 5 \quad and \quad g(x) = 2 - x^2$$

Evaluate the expressions.

c) $f(f(-4))$
d) $g(g(-4))$

Solution

c) $f\big(f(-4)\big) = f(4 \cdot (-4) - 5) = f(-21) = 4 \cdot (-21) - 5 = -89$
d) $g\big(g(-4)\big) = g(2 - (-4)^2) = g(-14) = 2 - (-14)^2 = 2 - 196 = -194$

Given $f(x) = 3x - 7, \quad g(x) = \frac{x}{3}$

Find $f \circ g, g \circ f, f \circ f$ and $g \circ g$ and their domain.

Solution

$f \circ g = 3 \cdot \frac{x}{3} - 7$ $= x - 7$ $Domain: (-\infty, \infty)$	$g \circ f = \frac{3x - 7}{3}$ $= x - \frac{7}{3}$ $Domain: (-\infty, \infty)$	$f \circ f = 3(3x - 7) - 7$ $= 9x - 21 - 7$ $= 9x - 28$ $Domain: (-\infty, \infty)$	$g \circ g = \frac{x/3}{3} = \frac{x}{9}$ $Domain: (-\infty, \infty)$

Given $f(x) = x^3 + 4, \quad g(x) = \sqrt[3]{x}$

Find $f \circ g, g \circ f, f \circ f$ and $g \circ g$ and their domain.

Solution

$f \circ g$ $= \left(\sqrt[3]{x}\right)^3 + 4$ $= x + 4$ $Domain:$ $(-\infty, \infty)$	$g \circ f$ $= \sqrt[3]{x^3 + 4}$ $Domain:$ $(-\infty, \infty)$	$f \circ f = (x^3 + 4)^3 + 4$ $= x^9 + 3 \cdot x^6 \cdot 4 + 3 \cdot x^3 \cdot 16 + 64 + 4$ $= x^9 + 12x^6 + 48x^3 + 68$ $Domain:$ $(-\infty, \infty)$	$g \circ g = \sqrt[3]{\sqrt[3]{x}}$ $= \left(x^{\frac{1}{3}}\right)^{\frac{1}{3}}$ $= x^{\frac{1}{9}}$ $Domain:$ $(-\infty, \infty)$

Given:

$$f(x) = \frac{8}{x} \qquad g(x) = x^3 \qquad h(x) = x^2 + 5$$

Find $f \circ g \circ h$.

Solution

$$f \circ g \circ h = f((x^2 + 5)^3)$$

$$= f(x^6 + 3 \cdot x^4 \cdot 5 + 3 \cdot x^2 \cdot 25 + 125)$$

$$= f(x^6 + 15x^4 + 75x^2 + 125)$$

$$= \frac{8}{x^6 + 15x^4 + 75x^2 + 125}$$