Week 5

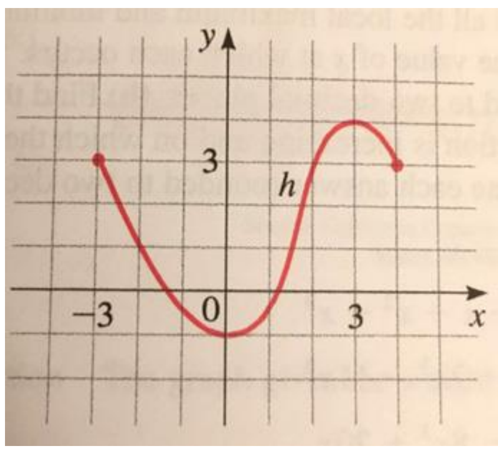
Sections 2.3, 2.4, 2.6

HW5: 8, 10, 16, 32, 44, 46 (p. 179-180)
8, 10, 12, 16 (p. 188)
8, 10, 12, 20, 30, 32, 34, 36, 46, 48 (p. 206-207)

Review Exercises

The graph of a function h is given.

a. Find $h(-2), h(0), h(2), h(3)$.
b. Find the domain and the range of h.
c. Find the values for x for which $h(x) = 3$
d. Find the values x for which $h(x) \leq 3$
e. Find the net change in h between $x = -3$ and $x = 3$.



Solution

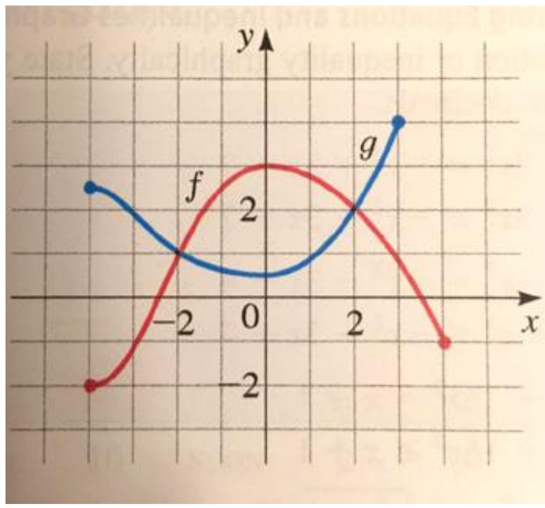
a. From the graph:

$$h(-2) = 1$$
$$h(0) = -1$$
$$h(2) = 3$$
$$h(3) = 4$$

b. $Domain = [-3, 4], \quad Range = [-1, 4]$
c. $h(x) = 3, when\ x = -3, and\ x = 2, and\ x = 4$
d. $h(x) \leq 3, when\ x\ is\ [-3, 2], and\ x = 4$
e. $h(3) = 4 \qquad h(-3) = 3$
   $Net\ Change = 4 - 3 = 1$

Graphs of the functions f and g are given.

a) Which is larger, $f(0)$ or $g(0)$?
b) Which is larger, $f(-3)$or $g(-3)$?
c) For which values of x is $f(x) = g(x)$?
d) Find the values of x for which $f(x) \leq g(x)$.
e) Find the values of x for which $f(x) > g(x)$.



Solution

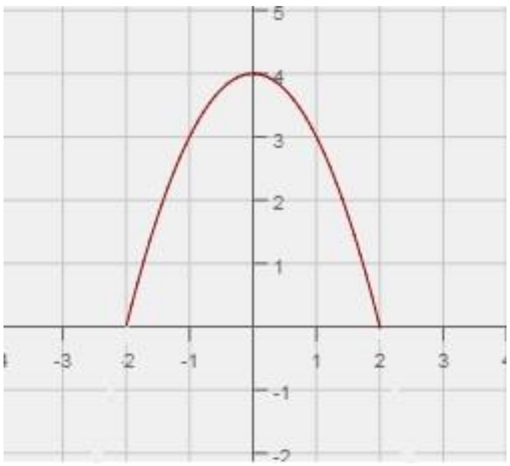
a) $f(0) = 3 \quad and \quad g(0) = {}^{1}/_{2}$, so $f(0)$ is larger.
b) $f(-3) = -1 \quad and \quad g(-3) = 2$, so $g(-3)$ is larger.
c) $f(x) = g(x)$ for $x = -2 \quad and \quad x = 2$
d) $f(x) \leq g(x)$ for the intervals: $[-4, -2] \;\; and \;\; [2, 3]$.
e) $f(x) > g(x)$ for the interval: $(-2, 2)$.

A function f is given.

$$f(x) = 4 - x^2, \quad -2 \leq x \leq 2$$

a) Sketch the graph of f.
b) Use the graph to find the domain and range of f.

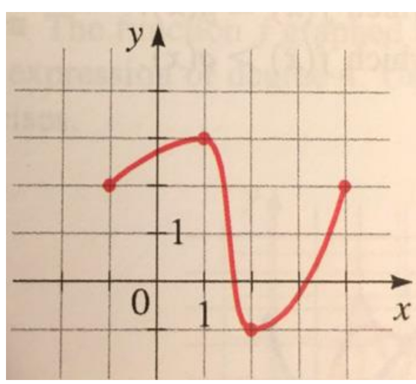
<u>Solution</u>

a) 

b) Domain: $[-2, 2]$, Range: $[0, 4]$

The graph of a function f is given. Use the graph to estimate the following.

a) The domain and range of f.
b) The intervals on which f is increasing and on which f is decreasing.



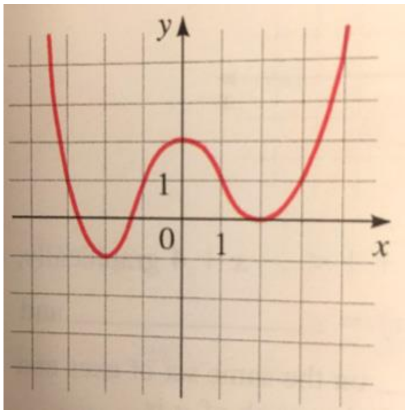
<u>Solution</u>

a) Domain: $[-1, 4]$, Range: $[-1, 3]$
b) The function is increasing on $(-1, 1)$ and $(2, 4)$
The function is decreasing on $(1, 2)$

The graph of a function f is given. Use the graph to estimate the following.

a) All maximum and minimum values of the function and the value of x at which each occurs.

b) The intervals on which the function is increasing and on which the function is decreasing.



<u>Solution</u>

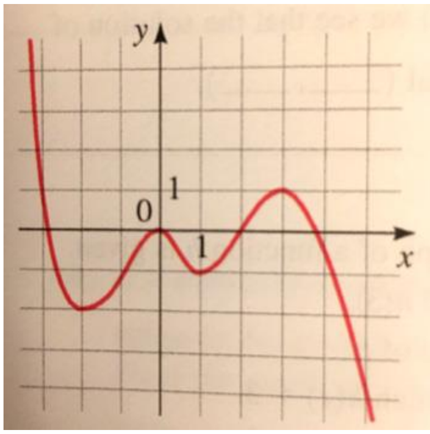
a) Local maximum: 2 at $x = 0$,
Local minimum: -1 at $x = -2$ and 0 at $x = 2$

b) The function is increasing on $(-2, 0)$ and $(2, \infty)$
The function is decreasing on $(-\infty, -2)$ and $(0, 2)$

The graph of a function f is given. Use the graph to estimate the following.

c) All the local maximum and minimum values of the function and the value of x at which each occurs.

d) The intervals on which the function is increasing and on which the function is decreasing.



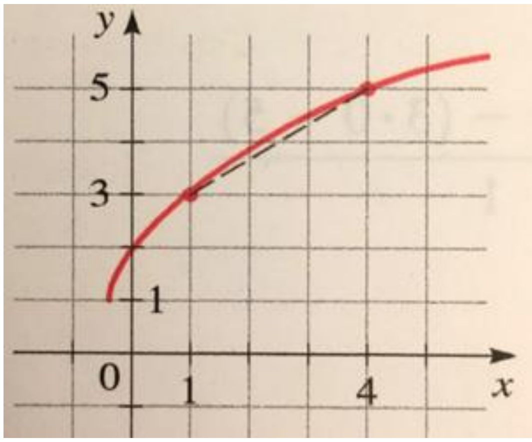
<u>Solution</u>

c) Local maximum: 0 at $x = 0$, and 1 at $x = 3$
Local minimum: -2 at $x = -2$ and -1 at $x = 1$

d) The function is increasing on $(-2, 0)$ and $(1, 3)$
The function is decreasing on $(-\infty, -2)$, $(0, 1)$, and $(3, \infty)$

The graph of a function is given. Determine:

a) The net change
b) The average rate of change between the indicated points of the graph.



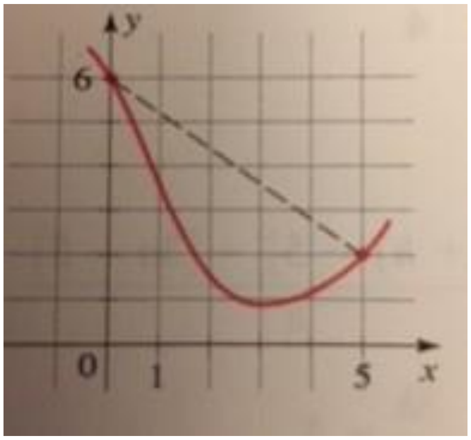
Solution

a) $f(4) - f(1) = 5 - 3 = 2$
b) Use the points $(1, 3)$ and $(4, 5)$

$$Average\ rate\ of\ change = \frac{5-3}{4-1} = \frac{2}{3}$$

The graph of a function is given. Determine:

c) The net change
d) The average rate of change between the indicated points of the graph.



Solution

c) $f(5) - f(0) = 2 - 6 = -4$
d) Use the points $(0, 6)$ and $(5, 2)$

$$Average\ rate\ of\ change = \frac{2-6}{5-0} = \frac{-4}{5}$$

A function is given. Determine:

a) The net change
b) The average rate of change between the given values of the variables.

$$f(x) = 3x - 2 \qquad x = 2, \qquad x = 3$$

<u>Solution</u>

a) The net change:

$$f(3) - f(2) = (3 \cdot 3 - 2) - (3 \cdot 2 - 2)$$

$$= (9 - 2) - (6 - 2)$$

$$= 7 - 4$$

$$= 3$$

b) The average rate of change:

$$\frac{f(3) - f(2)}{3 - 2} = \frac{3}{1} = 3$$

A function is given. Determine:

c) The net change
d) The average rate of change between the given values of the variables.

$$f(x) = 1 - 4x^2 \qquad x = -1, \qquad x = 0$$

<u>Solution</u>

c) The net change:

$$f(0) - f(-1) = (1 - 4 \cdot 0^2) - (1 - 4 \cdot (-1)^2)$$

$$= (1 - 0) - (1 - 4)$$

$$= 1 - (-3)$$

$$= 4$$

d) The average rate of change:

$$\frac{f(0) - f(-1)}{0 - (-1)} = \frac{4}{1} = 4$$

Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

a) $f(x+2)$
b) $f(x)+3$

Solution

a) The graph of $f(x+2)$ is obtained by shifting the graph of $f(x)$ to the left 2 units.
b) The graph of $f(x)+3$ is obtained by shifting the graph of $f(x)$ upward 3 units.

Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

c) $-f(x)$
d) $\frac{1}{5}f(x)$

Solution

c) The graph of $-f(x)$ is obtained by reflecting the graph of $f(x)$ about the x-axis.
d) The graph of $\frac{1}{5}f(x)$ is obtained by shrinking the graph of $f(x)$ vertically by a factor of $\frac{1}{5}$.

Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

e) $f(x+2)+4$
f) $f(x-8)-5$

Solution

e) The graph of $f(x+2)+4$ is obtained by shifting the graph of $f(x)$ to the left 2 units and upward 4 units.
f) The graph of $f(x-8)-5$ is obtained by shifting the graph of $f(x)$ to the right 8 units and downward 5 units.

Explain how the graph of g is obtained from the graph of f.

a) $f(x) = x^4, \ \ g(x) = (x-5)^4$
b) $f(x) = x^4, \ \ g(x) = x^4 - 5$

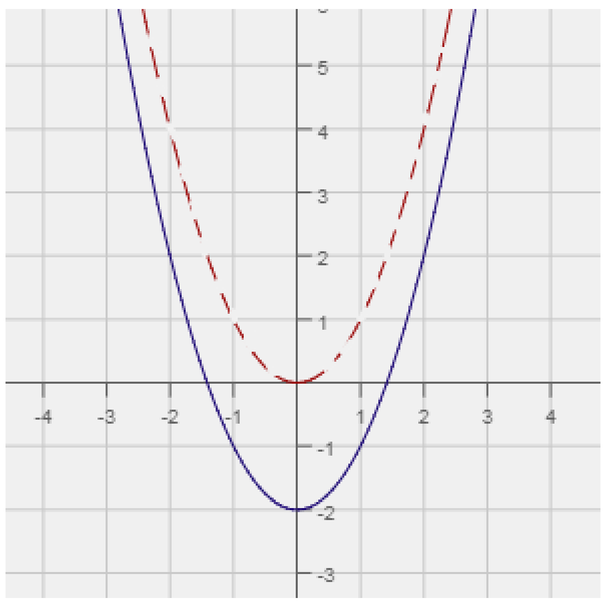
Solution

a) The graph of $g(x) = (x-5)^4$ is obtained by shifting the graph of $f(x)$ to the right 5 units.
b) The graph of $g(x) = x^4 - 5$ is obtained by shifting the graph of $f(x)$ downward 5 units.

Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = x^2 - 2$$

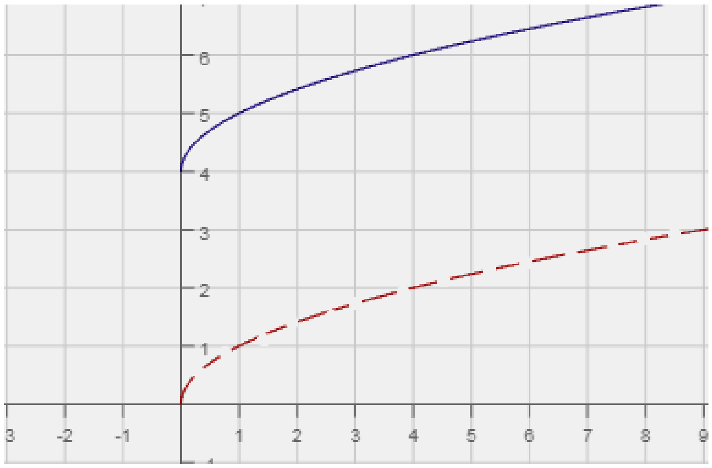
Solution



Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = \sqrt{x} + 4$$

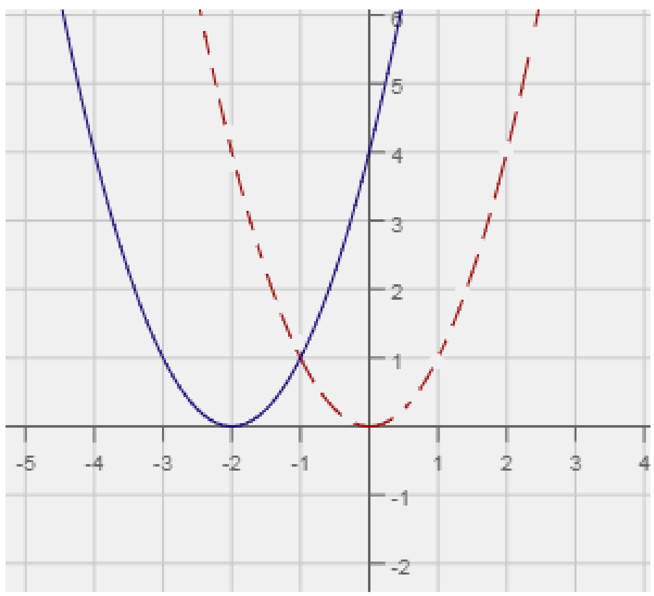
Solution



Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = (x+2)^2$$

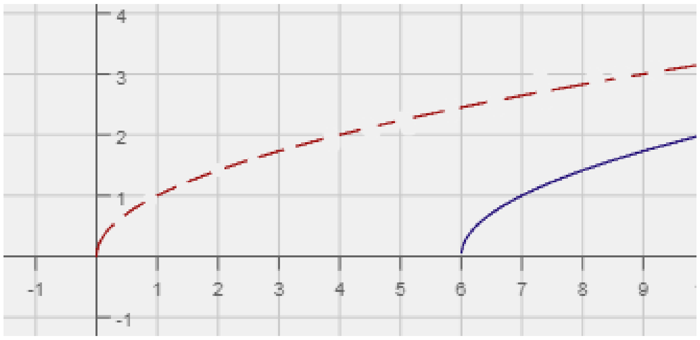
Solution



Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = \sqrt{x-6}$$

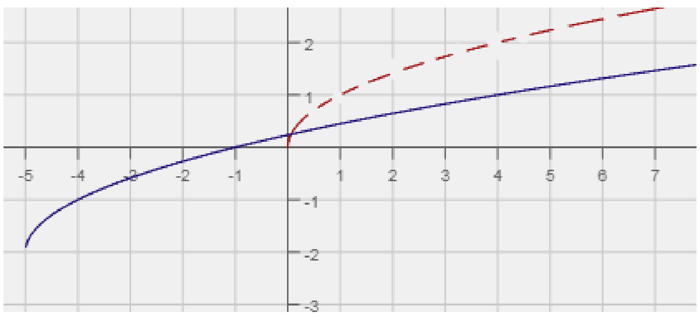
Solution



Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = \sqrt{x+5} - 2$$

Solution



Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$$f(x) = 1 - \sqrt{x+5}$$

Solution