Week 3

Sections 1.6, 1.8, 1.9

HW3: 8, 22, 24, 28, 30, 34, 40, 64 (p. 64)
16, 18, 30, 36, 38, 60 (p. 89)
28, 36, 60, 68, 88, 90 (p. 102-104)

Review Exercises
Find the real and the imaginary parts of the complex number. $$3 + 4i$$ Solution $$3 \text{ is the real part}$$ $$4 \text{ is the imaginary part}$$
Evaluate the sum and write the result in the form $a + bi$. $$(2 + 7i) + \left(-8 + \frac{1}{2}i\right)$$ Solution Combine the real parts together and combine the imaginary parts together. $$(2 + 7i) + \left(-8 + \frac{1}{2}i\right)$$ $$= -6 + \frac{15}{2}i$$
Evaluate the difference and write the result in the form $a + bi$. $$(-3 + 2i) - (-5 + 9i)$$ Solution Distribute the negative symbol in front of the second parenthesis. Combine the real parts together and combine the imaginary parts together. $$(-3 + 2i) - (-5 + 9i)$$ $$= -3 + 2i + 5 - 9i$$ $$= 2 - 7i$$

Evaluate the product and write the result in the form $a + bi$.

$$-5(4 + 6i)$$

Solution

Use distributive property to remove the parentheses.

$$-5(4 + 6i)$$

$$= -20 - 30i$$

Evaluate the product and write the result in the form $a + bi$.

$$(7 - i)(4 + 2i)$$

Solution

Use FOIL.

Replace i^2 with -1.

Combine the real parts together and combine the imaginary parts together.

$$(7 - i)(4 + 2i)$$

$$= 28 + 14i - 4i - 2i^2$$

$$= 28 + 14i - 4i - 2(-1)$$

$$= 28 + 14i - 4i + 2$$

$$= 30 + 10i$$

Evaluate the product and write the result in the form $a + bi$.

$$(2 + 5i)(2 - 5i)$$

Solution

$2 + 5i$ and $2 - 5i$ are complex conjugates.

Use the formula $(a + bi)(a - bi) = a^2 + b^2$.

$$(2 + 5i)(2 - 5i)$$

$$= 2^2 + 5^2$$

$$= 4 + 25$$

$$= 29$$

Evaluate the quotient and write the result in the form $a + bi$.

$$\frac{2-3i}{1-2i}$$

Solution

Multiply the top and the bottom by the conjugate of the denominator, which is $1 + 2i$. On the top use FOIL. On the bottom use the formula $(a + bi)(a - bi) = a^2 + b^2$. Replace i^2 with -1. Combine the real parts together and combine the imaginary parts together.

$$\frac{2-3i}{1-2i}$$

$$= \frac{(2-3i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{2+4i-3i-6i^2}{1^2+2^2}$$

$$= \frac{2+i+6}{5}$$

$$= \frac{8+i}{5}$$

$$= \frac{8}{5} + \frac{1}{5}i$$

Find all the solutions of the equation and express them in the form $a + bi$.

$$x^2 - 6x + 10 = 0$$

Solution

$$a = 1 \qquad b = -6 \qquad c = 10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{-4}}{2}$$

$$= \frac{6 \pm 2i}{2}$$

$$= 3 \pm i$$

Solve the linear inequality. Express the solution using interval notation and graph the solution set.

$$2x - 3 < -15$$

Solution

$$2x - 3 < -15$$
$$\quad +3 \quad\; +3$$

$$2x < -12$$

$$\frac{2x}{2} < \frac{-12}{2}$$

$$x < -6$$

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$-\infty$ $\quad$ -6 $\quad$ 0 $\quad$ ∞

The solution set is $(-\infty, -6)$.

Solve the linear inequality. Express the solution using interval notation and graph the solution set.

$$4 - 3x \leq -14$$

Solution

Solve for x, flip the inequality symbol when dividing by a negative.

$$4 - 3x \leq -14$$
$$-4 \qquad\quad -4$$

$$-3x \leq -18$$

$$\frac{-3x}{-3} \geq \frac{-18}{-3}$$

$$x \geq 6$$

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$-\infty$ $\quad$ 0 $\quad$ 6 $\quad$ ∞

The solution set is $[6, \infty)$.

Solve the linear inequality. Express the solution using interval notation and graph the solution set.

$$-7 \leq 5x - 2 < 8$$

<u>Solution</u>

$$\begin{array}{l} -7 \leq 5x - 2 < 8 \\ +2 \qquad\quad +2 \quad +2 \end{array}$$

$$-5 \leq 5x < 10$$

$$\frac{-5}{5} \leq \frac{5x}{5} < \frac{10}{5}$$

$$-1 \leq x < 2$$

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                 −1       0               2
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The solution set is $[-1, 2)$.

Solve the linear inequality. Express the solution using interval notation and graph the solution set.

$$-\frac{1}{2} \leq \frac{3-2x}{3} < \frac{1}{5}$$

<u>Solution</u>

$$LCD = 30$$

Multiply all three parts of the inequality by the LCD.

$$\left(-\frac{1}{2}\right) \cdot 30 \leq \left(\frac{3-2x}{3}\right) \cdot 30 < \left(\frac{1}{5}\right) \cdot 30$$

$$-15 \leq 10(3-2x) < 6$$

$$-15 \leq 30 - 20x < 6$$

$$-30 \quad -30 \qquad\quad -30$$

$$-45 \leq -20x < -24$$

$$\frac{-45}{-20} \geq \frac{-20x}{-20} > \frac{-24}{-20}$$

$$\frac{9}{4} \geq x > \frac{6}{5}$$

$$\frac{6}{5} < x \leq \frac{9}{4}$$

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<------------(------------------------]------------>
            6/5                      9/4
```

The solution set is $\left(\frac{6}{5}, \frac{9}{4}\right]$.

Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$$(x+2)(x-3) \geq 0$$

<u>Solution</u>

Find the values of x for which the factors of the left-side are 0.

$$x+2=0 \quad or \quad x-3=0$$
$$x=-2 \quad or \quad x=3$$

Set up the number line and choose test points in each interval.
Find the intervals for which the left-side of the inequality is positive or zero (≥).

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−∞          − 3          − 2          0            3           4              ∞
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Test Point -3	Test Point 0	Test Point 4
$(-3+2)(-3-3)$	$(0+2)(0-3)$	$(4+2)(4-3)$
$=(-1)(-6)$	$=(2)(-3)$	$=(6)(1)$
$=6$	$=-6$	$=6$
Positive	**Negative**	**Positive**

The solution set is $(-\infty,-2] \cup [3.\infty)$

Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$$\frac{5x-10}{x+3}<0$$

<u>Solution</u>

Find the values of x for which the numerator and the denominator on the left-side are 0.

$$\begin{aligned} 5x-10=0 \quad &or \quad x+3=0 \\ 5x=10 \quad &or \quad x=-3 \\ x=2 \quad &or \quad x=-3 \end{aligned}$$

Set up the number line and choose test points in each interval.

Find the intervals for which the left-side of the inequality is negative ($<$).

$-\infty$ -4 -3 0 2 3 ∞

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Test Point -4	Test Point 0	Test Point 3
$\frac{5(-4)-10}{-4+3}$	$\frac{5\cdot 0-10}{0+3}$	$\frac{5\cdot 3-10}{3+3}$
$=\frac{-30}{-1}$	$=\frac{-10}{3}$	$=\frac{5}{6}$
$=30$	$=-\frac{10}{3}$	**Positive**
Positive	**Negative**	

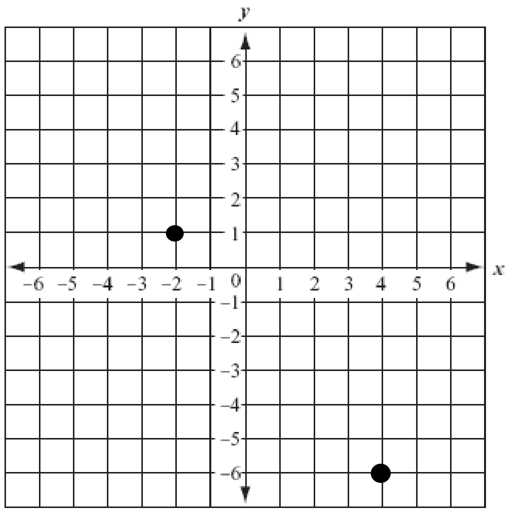
The solution set is $(-3, 2)$

The pair of points $(-2, 1)$ and $(4, -6)$ is given.

a. Plot the points in the coordinate plane.
b. Find the distance between them.
c. Find the midpoint of the segment that joins them.

Solution

a.



b.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-2))^2 + (-6 - 1)^2} \\ &= \sqrt{6^2 + (-7)^2} \\ &= \sqrt{36 + 49} \\ &= \sqrt{85} \end{aligned}$$

c.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2 + 4}{2}, \frac{1 + (-6)}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-5}{2}\right)$$

$$= \left(1, -\frac{5}{2}\right)$$

Which of the points $A(6,7)$ or $B(-5,8)$ is closer to point $C(3,2)$.

Solution

$$d(AC) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-6)^2 + (2-7)^2}$$
$$= \sqrt{(-3)^2 + (-5)^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$
$$\approx 5.8$$

$$d(BC) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-(-5))^2 + (2-8)^2}$$
$$= \sqrt{8^2 + (-6)^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100}$$
$$= 10$$

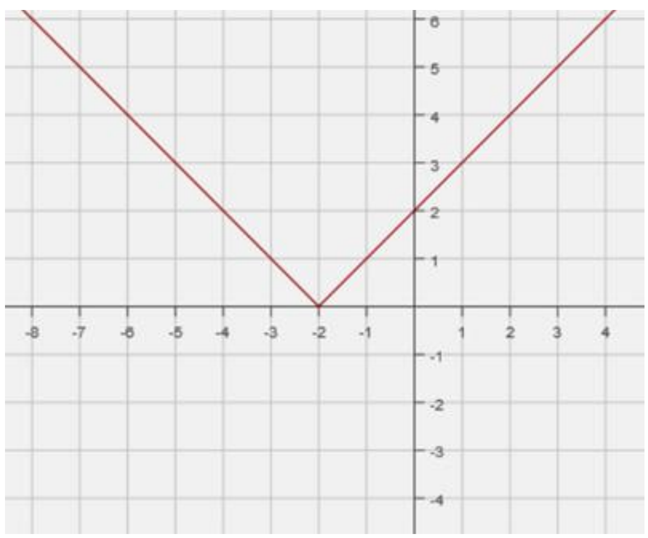
The point A is closer to point C.

Make a table of values, and sketch the graph of the equation.

$$y = |x + 2|$$

Solution

x	y
-3	1
-2	0
-1	1
0	2
1	3
2	4



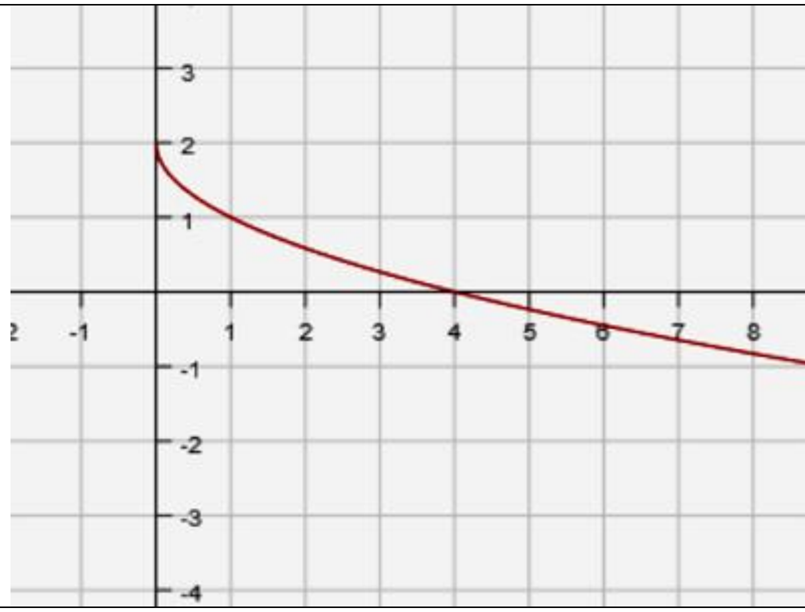
Make a table of values, and sketch the graph of the equation. Find the x- and y-intercepts, and test for symmetry.

a. $y = 2 - \sqrt{x}$
b. $x = |y|$

Solution

a.

x	y
0	2
1	1
4	0
9	-1

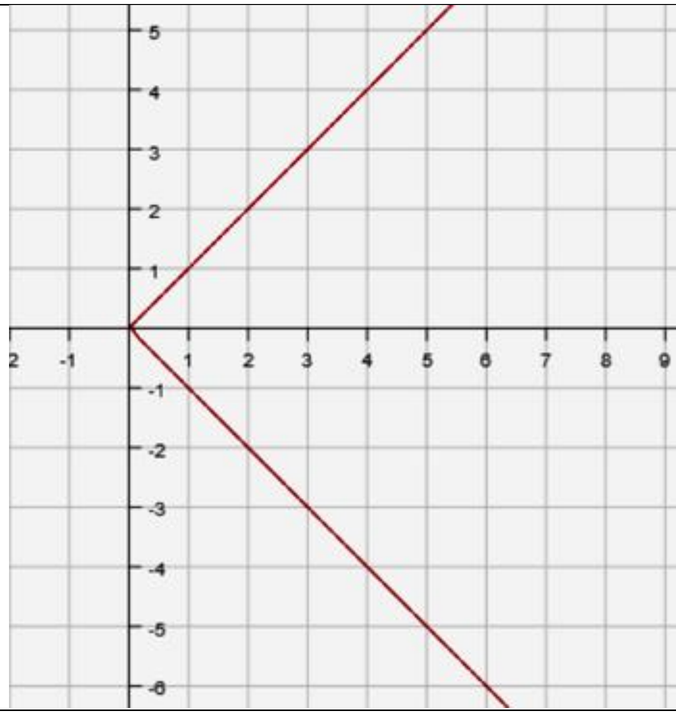


$$x - intercept = 4$$
$$y - intercept = 2$$

$x - axis\ symmetry$:	$y - axis\ symmetry$:	$Origin\ Symmetry$:
$-y = 3 - \sqrt{x}$ is not the same as $y = 3 - \sqrt{x}$	$y = 3 - \sqrt{-x}$ is not the same as $y = 3 - \sqrt{x}$	$-y = 3 - \sqrt{-x}$ is not the same as $y = 3 - \sqrt{x}$
The graph is not symmetric with respect to x-axis.	The graph is not symmetric with respect to y-axis.	The graph is not symmetric with respect to the origin.

b.

x	y
2	-2
1	-1
0	0
1	1
2	2



$x-intercept = 0$
$y-intercept = 0$

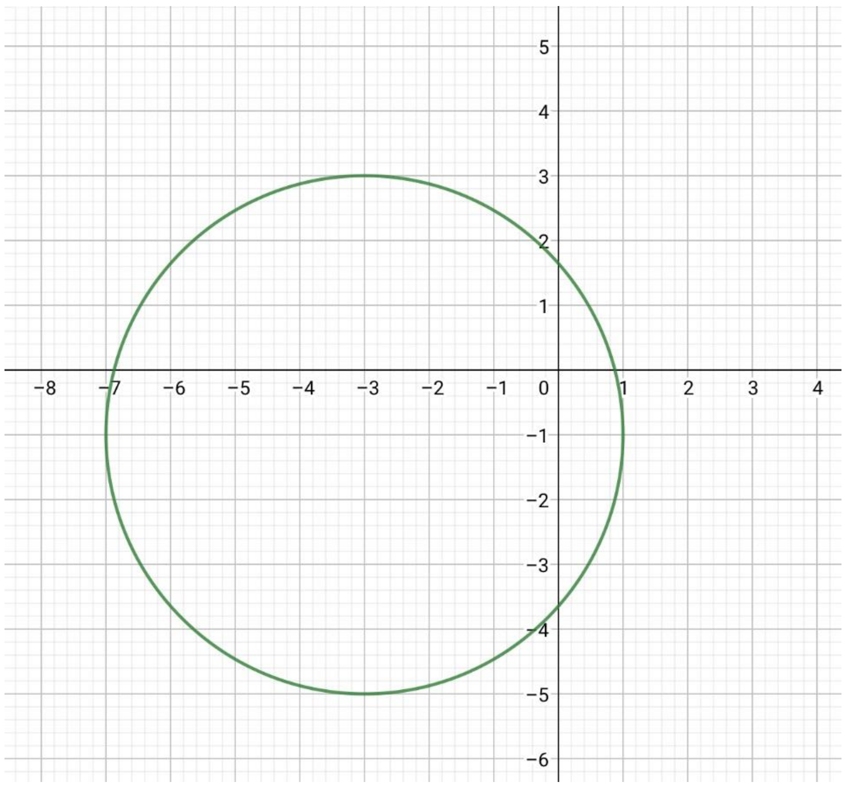
$x-axis\ symmetry$:	$y-axis\ symmetry$:	$Origin\ Symmetry$:
$x = \|-y\|$ is the same as $x = \|y\|$ The graph <u>is</u> symmetric with respect to x-axis.	$-x = \|y\|$ is not the same as $x = \|y\|$ The graph <u>is not</u> symmetric with respect to y-axis.	$-x = \|-y\|$ is not the same as $x = \|y\|$ The graph <u>is not</u> symmetric with respect to the origin.

Find the center and radius of the circle and sketch its graph.

$$(x+3)^2 + (x+1)^2 = 16$$

<u>Solution</u>

The center is $(-3, -1)$.
The radius is $r = 4$.



Find the equation of the circle that satisfies the given conditions.

Center $(6, -4)$
Radius $r = 8$

<u>Solution</u>

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-6)^2 + (y-(-4))^2 = 8^2$$

$$(x-6)^2 + (y+4)^2 = 64$$