Week 2

Sections 1.3, 1.4, 1.5

HW2: 18, 20, 30, 34, 38, 66, 70, 86 (p. 33-34)
10, 14, 20, 30, 44, 50 (p. 42-43)
18, 20, 48, 54, 70, 94 (p. 56-57)

Review Exercises

Find the difference of the polynomials.

$$(5x^2 - 4x + 5) - (3x^2 + 2x - 7)$$

Solution

Distribute the negative sign in front of the second parenthesis and combine like terms.

$$\begin{aligned}&(5x^2 - 4x + 5) - (3x^2 + 2x - 7)\\ &= 5x^2 - 4x + 5 - 3x^2 - 2x + 7\\ &= 2x^2 - 6x + 12\end{aligned}$$

Find the sum of the polynomials.

$$5(x + 3) + 2(x - 4)$$

Solution

Use distributive property to remove both parentheses and combine like terms.

$$\begin{aligned}&5(x + 3) + 2(x - 4)\\ &= 5x + 15 + 2x - 8\\ &= 7x + 7\end{aligned}$$

Multiply using FOIL.

$$(3x + 7y)(2x - y)$$

Solution

$$\begin{aligned}&(3x + 7y)(2x - y)\\ &= 6x^2 - 3xy + 14xy - 7y^2\\ &= 6x^2 + 11xy - 7y^2\end{aligned}$$

Multiply using special formulas.

$$(3x + 5y)^2$$

Solution

Use the formula $(a + b)^2 = a^2 + 2ab + b^2$.

$$(3x + 5y)^2 = 9x^2 + 30xy + 25y^2$$

Multiply using special formulas.

$$(4 + x)(4 - x)$$

Solution

Use the formula $(a + b)(a - b) = a^2 - b^2$.

$$(4 + x)(4 - x)$$
$$= 16 - x^2$$

Factor out the common factor.

$$(x + 3)^2 - 4(x + 3)$$

Solution

Start by factoring out the greatest common factor, which is $(x + 3)$.

$$(x + 3)^2 - 4(x + 3)$$
$$= (x + 3)(\boldsymbol{x + 3}) - 4(\boldsymbol{x + 3})$$
$$= (\boldsymbol{x + 3})(x + 3 - 4)$$
$$= (x + 3)(x - 1)$$

Factor the trinomial.

$$x^2 + 2x - 8$$

Solution

$$Sum = 2$$
$$Product = -8$$
$$The\ numbers\ are\ 4\ and\ -2$$

$$x^2 + 2x - 8$$
$$= (x + 4)(x - 2)$$

Factor the expression by grouping the terms.

$$x^3 + 4x^2 + x + 4$$

Solution

$$x^3 + 4x^2 + x + 4$$
$$= x^2(x + 4) + 1(x + 4)$$
$$= (x + 4)(x^2 + 1)$$

Find the domain of the expression.

$$\frac{7x^2 - 10}{2x + 10}$$

Solution

Find what number/numbers make the denominator 0, and exclude them from the domain.

$$2x + 10 \neq 0$$
$$-10 \quad -10$$

$$2x \neq -10$$

$$\frac{2x}{2} \neq \frac{-10}{2}$$

$$x \neq -5$$

The domain is $\{x | x \neq -5\}$.

Find the domain of the expression.

$$\frac{\sqrt{5x}}{x + 3}$$

Solution

Find what numbers make the expression inside the square root nonnegative and include these numbers in the domain.
Then find what number/numbers make the denominator 0, and exclude them from the domain.

$$5x \geq 0 \quad and \quad x + 3 \neq 0$$

$$\frac{5x}{5} \geq \frac{0}{5} \quad and \quad x + 3 \neq 0$$
$$-3 \quad -3$$

$$x \geq 0 \quad and \quad x \neq -3$$

The first condition $x \geq 0$ already includes the second one, that $x \neq -3$.

So, the domain is $\{x | x \geq 0\}$.

Simplify the rational expression.

$$\frac{x^2+5x+6}{x^2+8x+15}$$

Solution

Factor each trinomial and cancel the common factors.

$$\frac{x^2+5x+6}{x^2+8x+15}$$

$$=\frac{(x+2)\cancel{(x+3)}}{\cancel{(x+3)}(x+5)}$$

$$=\frac{x+2}{x+5}$$

Simplify the rational expression.

$$\frac{x^2+7x+12}{x^2+3x+2}\cdot\frac{x^2+5x+6}{2x+6}$$

Solution

Factor each polynomial and cancel the common factors before multiplying.

$$\frac{x^2+7x+12}{x^2+3x+2}\cdot\frac{x^2+5x+6}{2x+6}$$

$$=\frac{(x+3)(x+4)}{(x+1)\cancel{(x+2)}}\cdot\frac{\cancel{(x+2)(x+3)}}{2\cancel{(x+3)}}$$

$$=\frac{(x+3)(x+4)}{2(x+3)}$$

Perform the subtraction and simplify.

$$\frac{3}{x+1}-\frac{1}{x+2}$$

Solution

The Least Common Denominator is: $(x+1)(x+2)$

$$\frac{3}{x+1}-\frac{1}{x+2}$$

$$=\frac{3(x+2)}{(x+1)(x+2)}-\frac{1(x+1)}{(x+1)(x+2)}$$

$$=\frac{3x+6-x-1}{(x+1)(x+2)}$$

$$=\frac{2x+5}{(x+1)(x+2)}$$

Perform the addition and simplify.

$$\frac{1}{x}+\frac{3}{x^2}+\frac{2}{x^3}$$

Solution

The Least Common Denominator is x^3.

$$\frac{1}{x}+\frac{3}{x^2}+\frac{2}{x^3}$$

$$=\frac{1\cdot x^2}{x\cdot x^2}+\frac{3\cdot x}{x^2\cdot x}+\frac{2}{x^3}$$

$$=\frac{x^2+3x+2}{x^3}$$

Solve the equation.

$$7x-5=3+5x$$

Solution

$$7x-5=3+5x$$
$$7x-5x=3+5$$
$$2x=8$$
$$\frac{2x}{2}=\frac{8}{2}$$

$$x=4$$

Solve the equation.

$$\frac{1}{3}x - 2 = \frac{5}{6}x + 7$$

Solution

$$LCD = 6$$

Multiply both sides by the Least Common Denominator.

$$6\left(\frac{1}{3}x - 2\right) = 6\left(\frac{5}{6}x + 7\right)$$

$$2x - 12 = 5x + 42$$
$$-12 - 42 = 5x - 2x$$
$$-54 = 3x$$

$$\frac{-54}{3} = \frac{3x}{3}$$

$$-18 = x$$

Find all the real solutions of the equation by factoring.

$$x^2 + 7x + 12 = 0$$

Solution

Factor the trinomial, then set each factor equal to 0, and solve the simple equations.

$$Sum = 7$$
$$Product = 12$$

$$(x + 3)(x + 4) = 0$$
$$x + 3 = 0 \quad or \quad x + 4 = 0$$
$$-3 \ \ -3 \qquad\qquad -4 \ \ -4$$

$$x = -3 \quad or \quad x = -4$$

Find all the real solutions of the equation by factoring.

$$2x^2 - 50 = 0$$

Solution

Factor the binomial, then set each factor equal to 0, and solve the simple equations.

$$2(x^2 - 25) = 0$$
$$2(x + 5)(x - 5) = 0$$
$$x + 5 = 0 \quad or \quad x - 5 = 0$$
$$-5 \ \ -5 \qquad\qquad +5 \ \ +5$$

$$x = -5 \quad or \quad x = 5$$

Find all the real solutions of the quadratic equation.

$$3x^2 + 6x - 5 = 0$$

Solution

$$a = 3 \quad b = 6 \quad c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$

$$= \frac{-6 \pm \sqrt{96}}{6}$$

$$= \frac{-6 \pm \sqrt{16 \cdot 6}}{6}$$

$$= \frac{-6 \pm 4\sqrt{6}}{6}$$

$$= \frac{2(-3 \pm 2\sqrt{6})}{6}$$

$$= \frac{-3 \pm 2\sqrt{6}}{3}$$

Find all the real solutions of the equation.

$$\sqrt{4x - 3} = 5$$

Solution

Raise both sides of the equation to the second power to remove the radical symbol.

$$\left(\sqrt{4x - 3}\right)^2 = (5)^2$$

$$4x - 3 = 25$$
$$+3 \quad +3$$

$$4x = 28$$
$$\frac{4x}{4} = \frac{28}{4}$$

$$x = 7$$