Week 16

Sections 13.1, 13.2

HW: 16

4, 6, 10, 12, 14, 16, 18, 20, 30, 32 (p. 905)
6, 8, 10, 14, 18, 20, 22, 24, 28, 38 (p. 913)

Review Exercises

Example

Estimate the value of the limit by making a table of values. Check with a graph.

$$\lim_{x\to 4}\frac{x^2-2x-8}{x-4}$$

Solution

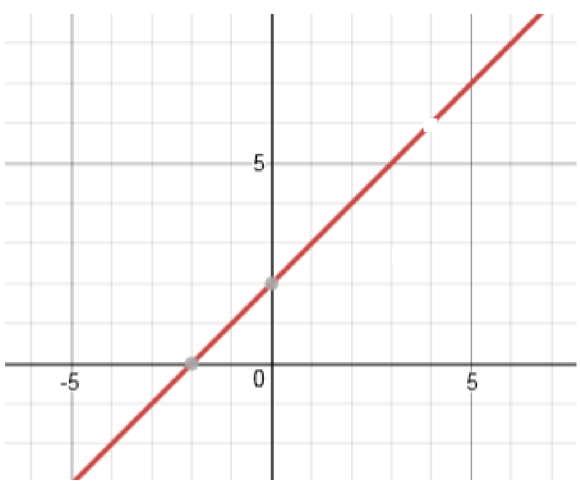
Create a table with values of x to the left and to the right of 4.

x	3.9	3.99	3.999	4.001	4.01	4.1
$\frac{x^2-2x-8}{x-4}$	5.9	5.99	5.999	6.001	6.01	6.1

To graph, simplify:

$$\frac{x^2-2x-8}{x-4}=\frac{(x+2)(x-4)}{x-4}=x+2 \qquad x\neq 4$$

Use the slope $m=1$ and $y-intercept=2$ to make the graph.



From the graph, the limit appears to be 6.

Example

Complete the table of values (to 5 decimal places), and use the table to estimate the limit.

$$\lim_{x \to 4} \frac{x - 4}{x^2 - 6x + 8}$$

Solution

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.52632	0.50251	0.50025	0.49975	0.49751	0.47619

$$\lim_{x \to 4} \frac{x - 4}{x^2 - 6x + 8} = 0.5$$

Example

Complete the table of values (to 5 decimal places), and use the table to estimate the limit.

$$\lim_{x \to 0^+} x \log x$$

Solution

x	0.1	0.01	0.001	0.0001	0.00001
$f(x)$	-0.10000	-0.02000	-0.00300	-0.00040	-0.00005

$$\lim_{x \to 0^+} x \log x = 0$$

Example

Use a table of values to estimate the value of the limit.

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$$

Solution

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x^3 - 8}{x^2 - 4}$	2.92564	2.99251	2.99925	3.00075	3.00751	3.07561

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = 3$$

No need to graph, if you do not have a graphing calculator.

Example

Use a table of values to estimate the value of the limit.

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$$

Solution

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sqrt{x+1} - 1}{x}$	0.51317	0.50126	0.50013	0.49988	0.49876	0.48809

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} = 0.5$$

No need to graph, if you do not have a graphing calculator.

Example

Use a table of values to estimate the value of the limit.

$$\lim_{x\to 0} \frac{\tan 3x}{\tan 5x}$$

Solution

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\tan 3x}{\tan 5x}$	0.56624	0.59968	0.60000	0.60000	0.59968	0.56624

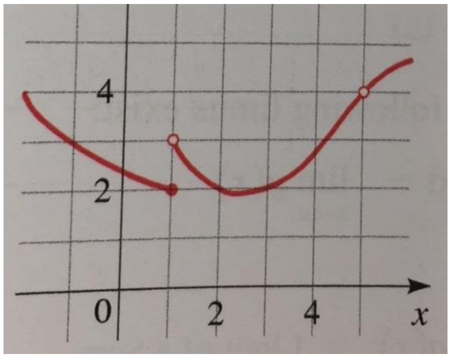
$$\lim_{x\to 0} \frac{\tan 3x}{\tan 5x} = 0.60000$$

No need to graph, if you do not have a graphing calculator.

Example

For the function whose graph is given, state the value of the given quantity if it exists. If it does not exist, explain why.

a. $\lim_{x\to 1^-} f(x)$
b. $\lim_{x\to 1^+} f(x)$
c. $\lim_{x\to 1} f(x)$
d. $\lim_{x\to 5} f(x)$
e. $f(5)$



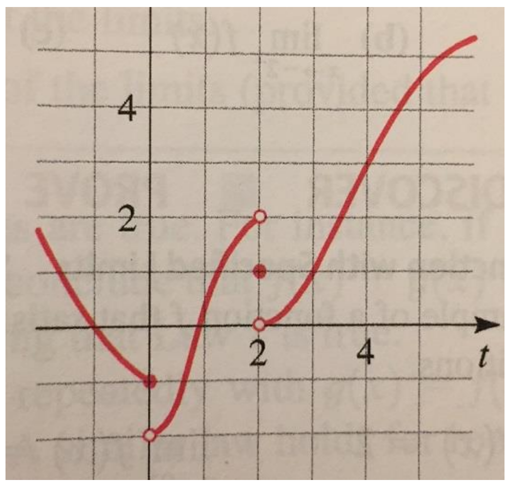
Solution

a. $\lim_{x\to 1^-} f(x) = 2$
b. $\lim_{x\to 1^+} f(x) = 3$
c. $\lim_{x\to 1} f(x)$ *does not exist, because* $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$
d. $\lim_{x\to 5} f(x) = 4$
e. $f(5)$ *is not defined.*

Example

For the function whose graph is given, state the value of the given quantity if it exists. If it does not exist, explain why.

a. $\lim_{t\to 0^-} f(t)$
b. $\lim_{t\to 0^+} f(t)$
c. $\lim_{t\to 0} f(t)$
d. $\lim_{t\to 2^-} f(t)$
e. $\lim_{t\to 2^+} f(t)$
f. $\lim_{t\to 2} f(t)$
g. $f(2)$
h. $\lim_{t\to 4} f(t)$



<u>Solution</u>

a. $\lim_{t\to 0^-} f(t) = -1$
b. $\lim_{t\to 0^+} f(t) = -2$
c. $\lim_{t\to 0} f(t)\ does\ not\ exist, because\ \lim_{t\to 0^-} f(t) \neq \lim_{t\to 0^+} f(t)$
d. $\lim_{t\to 2^-} f(t) = 2$
e. $\lim_{t\to 2^+} f(t) = 0$
f. $\lim_{t\to 2} f(t)\ does\ not\ exist, because\ \lim_{t\to 2^-} f(t) \neq \lim_{t\to 2^+} f(t)$
g. $f(2) = 1$
h. $\lim_{t\to 4} f(t) = 3$

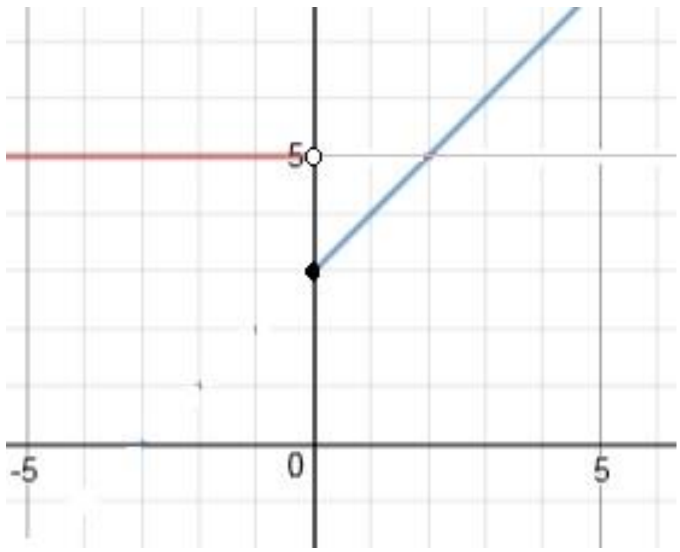
Example

Graph the piecewise-defined function and use the graph to find the values of the limits if they exist.

$$f(x) = \begin{cases} 5 & if\ x < 0 \\ x + 3 & if\ x \geq 0 \end{cases}$$

a. $\lim_{x \to 0^-} f(x)$
b. $\lim_{x \to 0^+} f(x)$
c. $\lim_{x \to 0} f(x)$

<u>Solution</u>



a. $\lim_{x \to 0^-} f(x) = 5$
b. $\lim_{x \to 0^+} f(x) = 3$
c. $\lim_{x \to 0} f(x)\ does\ not\ exist.$

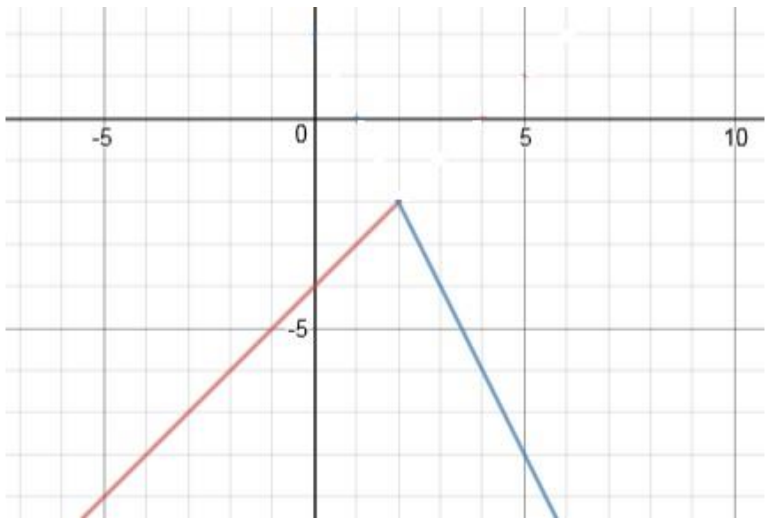
Example

Graph the piecewise-defined function and use the graph to find the values of the limits if they exist.

$$f(x) = \begin{cases} x - 4 & if\ x \le 2 \\ -2x + 2 & if\ x > 2 \end{cases}$$

a. $\lim_{x \to 2^-} f(x)$
b. $\lim_{x \to 2^+} f(x)$
c. $\lim_{x \to 2} f(x)$

<u>Solution</u>



a. $\lim_{x \to 2^-} f(x) = -2$
b. $\lim_{x \to 2^+} f(x) = -2$
c. $\lim_{x \to 2} f(x) = -2$

Example

Evaluate the limit and apply the appropriate Limit Law.

$$\lim_{x \to 0} 5$$

<u>Solution</u>

$$\lim_{x \to 0} 5 = 5, \qquad because\ \lim_{x \to a} c = c$$

Example

Evaluate the limit and apply the appropriate Limit Law.

$$\lim_{x\to 4}(2x-5)$$

Solution

$$\lim_{x\to 4}(2x-5) = \lim_{x\to 4}(2x) - \lim_{x\to 4}(-5)$$

$$= 2\lim_{x\to 4}(x) - 5$$

$$= 2\cdot 4 - 5$$
$$= 8 - 5$$
$$= 3$$

$$because$$
$$\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$
$$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$$
$$\lim_{x\to a} c = c$$

Example

Evaluate the limit and apply the appropriate Limit Law.

$$\lim_{x\to 0}(2x^3+4x^2-6)$$

Solution

$$\lim_{x\to 0}(2x^3+4x^2-6)$$

$$= \lim_{x\to 0}(2x^3) + \lim_{x\to 0}(4x^2) - \lim_{x\to 0}(6)$$

$$= 2\lim_{x\to 0}(x^3) + 4\lim_{x\to 0}(x^2) - \lim_{x\to 0}(6)$$

$$= 2\cdot 0^3 + 4\cdot 0^2 - 6$$

$$= -6$$

$$because$$
$$\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$
$$\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$
$$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$$
$$\lim_{x\to a} c = c, \qquad \lim_{x\to a} x = a, \qquad \lim_{x\to a} x^n = a^n$$

Example

Evaluate the limit and apply the appropriate Limit Law.

$$\lim_{x\to 5}(x-3)^4(x+1)$$

Solution

$$\lim_{x\to 5}(x-3)^4(x+1)$$

$$= (\lim_{x\to 5}(x-3)^4)\,(\lim_{x\to 5}(x+1))$$

$$= (\lim_{x\to 5}(x-3))^4\,(\lim_{x\to 5}(x+1))$$

$$= (\lim_{x\to 5}(x) - \lim_{x\to 5}(3))^4(\lim_{x\to 5}(x) + \lim_{x\to 5}(1))$$

$$= (5-3)^4(5+1)$$

$$= 2^4 \cdot 6$$

$$= 16 \cdot 6$$

$$= 96$$

$$because$$

$$\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\big(f(x)\big)^n = (\lim_{x\to a}\big(f(x)\big)^n$$

$$\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

$$\lim_{x\to a} c = c, \qquad \lim_{x\to a} x = a, \qquad \lim_{x\to a} x^n = a^n$$

Example

Evaluate the limit and apply the appropriate Limit Law.

$$\lim_{x\to -2}\sqrt{x^2+4x+5}$$

Solution

$$\lim_{x\to -2}\sqrt{x^2+4x+5}$$

$$=\sqrt{\lim_{x\to -2}(x^2+4x+5)}$$

$$=\sqrt{\lim_{x\to -2}(x^2)+\lim_{x\to -2}(4x)+\lim_{x\to -2}(5)}$$

$$=\sqrt{\lim_{x\to -2}(x^2)+4\lim_{x\to -2}(x)+\lim_{x\to -2}(5)}$$

$$=\sqrt{(-2)^2+4(-2)+5}$$

$$=\sqrt{4-8+5}$$

$$=\sqrt{1}$$

$$=1$$

$$because$$

$$\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a}f(x)}$$

$$\lim_{x\to a}(f(x)+g(x))=\lim_{x\to a}f(x)+\lim_{x\to a}g(x)$$

$$\lim_{x\to a}cf(x)=c\lim_{x\to a}f(x)$$

$$\lim_{x\to a}c=c,\qquad \lim_{x\to a}x=a,\qquad \lim_{x\to a}x^n=a^n$$

Example

Evaluate the limit if it exists.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2+2x-8}$$

Solution

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2+2x-8}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+4)}$$

$$=\lim_{x\to 2}\frac{(x+3)}{(x+4)}$$

$$=\frac{2+3}{2+4}$$

$$=\frac{5}{6}$$

Example

Evaluate the limit if it exists.

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}$$

Solution

To factor, use the formula $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\lim_{x\to 2}\frac{x^3-2^3}{x^2-2^2}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2}\frac{(x^2+2x+4)}{(x+2)}$$

$$=\frac{(2^2+2\cdot 2+4)}{(2+2)}$$

$$=\frac{12}{4}=3$$

Example

Evaluate the limit if it exists.

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$$

<u>Solution</u>

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x^2 - 9)(x^2 + 9)}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x - 3)(x + 3)(x^2 + 9)}{x - 3}$$

$$= \lim_{x \to 3} (x + 3)(x^2 + 9)$$

$$= (3 + 3)(3^2 + 9)$$

$$= 6 \cdot 18$$

$$= 10$$

Example

Evaluate the limit if it exists.

$$\lim_{x \to 0} \frac{\sqrt{4+x}-2}{x}$$

<u>Solution</u>

Multiply both the numerator and the denominator by the conjugate of $\sqrt{4+x}-2$, which is $\sqrt{4+x}+2$.

$$\lim_{x \to 0} \frac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\sqrt{4+x}+2)}$$

Use the formula $(a-b)(a+b)=a^2-b^2$ to remove the parentheses on the numerator.

$$\lim_{x \to 0} \frac{\left(\sqrt{4+x}\right)^2-2^2}{x(\sqrt{4+x}+2)}$$

$$= \lim_{x \to 0} \frac{4+x-4}{x(\sqrt{4+x}+2)}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{4+x}+2)}$$

$$= \lim_{x \to 0} \frac{1}{(\sqrt{4+x}+2)}$$

$$= \frac{1}{(\sqrt{4+0}+2)}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

Example

Evaluate the limit if it exists. If it does not exist, explain why.

$$\lim_{x \to -5^-} \frac{|x+5|}{x+5}$$

Solution

With the absolute value, when x approaches 5 from the left, the expression $|x+5|$ becomes $-(x+5)$.

$$\lim_{x \to -5^-} \frac{|x+5|}{x+5}$$

$$= \lim_{x \to -5^-} \frac{-(x+5)}{x+5}$$

$$= \lim_{x \to -5^-} \frac{-1(x+5)}{(x+5)}$$

$$= -1$$