Week 15

Sections 10.1

HW15: 6, 10, 16, 22, 24, 60, 62, 64, 66, 72 (p. 688-690)

Review Exercises
Example

Use substitution method to solve the system.

$$\begin{cases} x + 3y = 11 \\ x - 5y = -13 \end{cases}$$

<u>Solution</u>

Solve the second equation for x.
$x - 5y = -13$
$\quad +5y \quad\quad + 5y$

$x = 5y - 13$

In the first equation, replace x with $5y - 13$.

$5y - 13 + 3y = 11$

$8y - 13 = 11$
$\quad\quad +13 \quad + 13$

$8y = 24$

$\dfrac{8y}{8} = \dfrac{24}{8}$

$y = 3$

Substitute $y = 3$ in the first equation and find x.

$x + 3y = 11$
$x + 3 \cdot 3 = 11$
$x + 9 = 11$
$\quad -9 \quad - 9$
$x = 2$

The solution set is $(2, 3)$.

Example

Use the elimination method (addition method) to solve the system.

$$\begin{cases} 2x - 7y = 2 \\ 3x + y = -20 \end{cases}$$

<u>Solution</u>

Multiply the second equation by 7.

$$\begin{cases} 2x - 7y = 2 \\ 3x + y = -20 \end{cases} \quad (7)$$

$$\begin{cases} 2x - 7y = 2 \\ 21x + 7y = -140 \end{cases}$$

Add the equations.

$$\begin{cases} 2x - 7y = 2 \\ 21x + 7y = -140 \end{cases}$$

$$23x = -138$$

$$x = -6$$

Use any of the equations, for example, the first one, to substitute x.

$2x - 7y = 2$
$2(-6) - 7y = 2$
$-12 - 7y = 2$
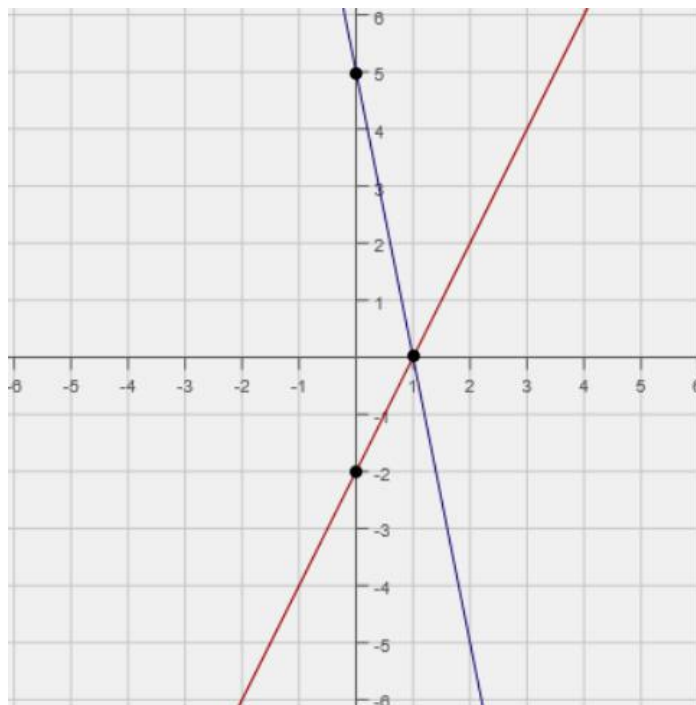$+12 \qquad + 12$
$-7y = 14$
$y = -2$

The solution is $(-6, -2)$.

Example

Solve the system by graphing.

$$\begin{cases} 2x - y = 2 \\ 5x + y = 5 \end{cases}$$

Solution

2x − y = 2		5x + y = 5	
$x - intercept$	$y - intercept$	$x - intercept$	$y - intercept$
$(\ ,0)$	$(0,\ )$	$(\ ,0)$	$(0,\ )$
$2x - 0 = 2$ $2x = 2$ $\dfrac{2x}{2} = \dfrac{2}{2}$ $x = 1$ $(\mathbf{1,0})$	$2 \cdot 0 - y = 2$ $-y = 2$ $\dfrac{-y}{-1} = \dfrac{2}{-1}$ $y = -2$ $(\mathbf{0,-2})$	$5x + 0 = 5$ $5x = 5$ $\dfrac{5x}{5} = \dfrac{5}{5}$ $x = 1$ $(\mathbf{1,0})$	$5 \cdot 0 + y = 5$ $y = 5$ $(\mathbf{0,5})$
Plot the intercepts and connect them with a line.		Plot the intercepts and connect them with a line.	

The point of intersection is $(1, 0)$, so, the solution is $(1, 0)$.

Example

Solve the system.

$$\begin{cases} x + 2y = 2 \\ -4x + 3y = 25 \end{cases}$$

<u>Solution</u>

Multiply the first equation by 4.

$$\begin{cases} x + 2y = 2 \quad (4) \\ -4x + 3y = 25 \end{cases}$$

$$\begin{cases} 4x + 8y = 8 \\ -4x + 3y = 25 \end{cases}$$

Add the equations.

$$\begin{cases} 4x + 8y = 8 \\ -4x + 3y = 25 \end{cases}$$

$$11y = 33$$

$$y = 3$$

Use any of the equations, for example, the first one, to replace y with 3.

$$x + 2y = 2$$

$$x + 2 \cdot 3 = 2$$

$$x + 6 = 2$$
$$ -6 \quad -6$$

$$x = -4$$

The solution is $(-4 , 3)$.

Example

Solve the system.

$$\begin{cases} 2x - 3y = 9 \\ 4x + 3y = 9 \end{cases}$$

<u>Solution</u>

Add the equations.

$$\begin{cases} 2x - 3y = 9 \\ 4x + 3y = 9 \end{cases}$$

$$6x = 18$$

$$x = 3$$

Use any of the equations, for example, the second one, to replace x with 3.

$$4x + 3y = 9$$

$$4 \cdot 3 + 3y = 9$$

$$12 + 3y = 9$$
$$-12 \quad\;\; -12$$

$$3y = -3$$

$$y = -1$$

The solution is $(3, -1)$.

Example

There are two numbers. The larger one is x and the smaller one is y. Twice the larger number plus the smaller number, equals four times their difference. The larger number is one more than twice the smaller number. Find the numbers.

Solution

$$\begin{cases} 2x + y = 4(x - y) \\ \quad x = 2y + 1 \end{cases}$$

$$\begin{cases} 2x + y = 4x - 4y \\ \quad x = 2y + 1 \end{cases}$$

In the first equation, replace x with $2y + 1$.

$$2(2y + 1) + y = 4(2y + 1) - 4y$$

$$4y + 2 + y = 8y + 4 - 4y$$

$$5y + 2 = 4y + 4$$

$$5y - 4y = 4 - 2$$

$$y = 2$$

$$x = 2y + 1$$
$$x = 2 \cdot 2 + 1$$
$$x = 5$$

The number are 5 and 2.

Example

1874 tickets were sold at an amusement park for a total of $21,356. If each child paid $9 and each adult paid $14, how many children bought tickets?

<u>Solution</u>

Let x be the number of children.
Let $1874 - x$ be the number of adults.

$9x$ is the cost of all the children tickets.
$14(1874 - x)$ is the cost of all the adult tickets.

$$9x + 14(1874 - x) = 21356$$

$$9x + 26236 - 14x = 21356$$ Use distributive property to remove the parentheses.

$$-5x + 26236 = 21356$$ Combine $9x - 14x = -5x$

$$\begin{array}{r} -5x + 26236 = 21356 \\ -26236 \quad -26236 \end{array}$$ Subtract 26236 on both sides.

$$-5x = -4880$$

$$\frac{-5x}{-5} = \frac{-4880}{-5}$$ Divide both sides by -5.

$$x = 976$$

976 children and 898 adults bought tickets.

Example

A grocer sells two types of apples, Fuji and Honeycrisp. One pound of Fuji costs $2, and one pound of Honeycrisp costs $3.25. If the grocer sold 10 pounds of apples for a total of $25, how many pounds of each type were sold?

<u>Solution</u>

Let x be the number of pounds of Fuji apples.
Let y be the number of pounds of Honeycrisp apples.

$$\begin{cases} x + y = 10 \\ 2x + 3.25y = 25 \end{cases}$$

$$\begin{cases} y = 10 - x \\ 2x + 3.25y = 25 \end{cases}$$

$$2x + 3.25(10 - x) = 25$$

$$2x + 32.5 - 3.25x = 25$$

$$-1.25x + 32.5 = 25$$

$$-1.25x = 25 - 32.5$$

$$-1.25x = -7.5$$

$$\frac{-1.25x}{-1.25} = \frac{-7.5}{-1.25}$$

$$x = 6$$

$$y = 10 - 6$$

$$y = 4$$

6 pounds of Fuji apples and 4 pounds of Honeycrisp apples must be sold.

Example

A plane flies with the wind, and it travels 800 miles in 5 hours. The return trip against the wind takes 8 hours. Find the speed of the plane in still air and the speed of the wind.

Solution

Let x be the speed of the plane in still air.
Let y be the speed of the wind.

	Rate	Time	Rate × Time = Distance
Trip with the wind.	$x + y$	5	$5(x + y) = 800$
Trip against the wind.	$x - y$	8	$8(x - y) = 800$

Solve the system:

$$\begin{cases} 5(x + y) = 800 \\ 8(x - y) = 800 \end{cases}$$

Divide both sides of the first equation by 5, and the both sides of the second equation by 8.

$$\begin{cases} 5(x + y) = 800 & \div 5 \\ 8(x - y) = 800 & \div 8 \end{cases}$$

$$\begin{cases} x + y = 160 \\ x - y = 100 \end{cases}$$
$$\text{--------------------}$$
$$2x = 260$$

$$\frac{2x}{2} = \frac{260}{2}$$

$$x = 130$$

Use $x + y = 160$ to find y.

$$130 + y = 160$$

$$y = 30$$

The plane's rate in still air is **130 mph**, and the rate of the wind is **30 mph**.

Example

A man invested into two accounts, one paying 7% interest per year and the other paying 9% interest per year. He invested three times as much money at 7% than he invested at 9%. His annual interest is $3,600. Determine the amount of money he invested in each account.

<u>Solution</u>

	Amount in dollars	The amount of interest from each account	Total interest
At 7%	x	$0.07x$	$3,600
At 9%	y	$0.09y$	

Write the system:

$$\begin{cases} x = 3y \\ 0.07x + 0.09y = 3,600 \end{cases}$$

In the second equation, replace x with $3y$.

$$0.07(3y) + 0.09y = 3,600$$

$$0.21y + 0.09y = 3,600$$

$$0.3y = 3,600$$

$$\frac{0.3y}{0.3} = \frac{3,600}{0.3}$$

$$y = 12,000$$

$$x = 3y = 3 \cdot 12,000$$

$$x = 36,000$$

The man invested $36,000 and 7%, and $12,000 at 9%.