Week 11

Sections 5.1

HW11: 4, 6, 8, 24, 28, 32, 38, 42, 46, 50 (p. 407-408)

Review Exercises

Show that the point is on the circle.

$$\left(\frac{3}{5}, -\frac{4}{5}\right)$$

Solution

We have to show that $x^2 + y^2 = 1$.

$$\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2 = 1$$

$$\frac{9}{25} + \frac{16}{25} = 1$$

$$\frac{25}{25} = 1$$

Show that the point is on the circle.

$$\left(-\frac{2}{7}, -\frac{3\sqrt{5}}{7}\right)$$

Solution

We have to show that $x^2 + y^2 = 1$.

$$\left(-\frac{2}{7}\right)^2 + \left(-\frac{3\sqrt{5}}{7}\right)^2 = 1$$

$$\frac{4}{49} + \frac{9 \cdot 5}{49} = 1$$

$$\frac{4}{49} + \frac{45}{49} = 1$$

$$\frac{49}{49} = 1$$

Show that the point is on the circle.

$$\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$$

Solution

We have to show that $x^2 + y^2 = 1$.

$$\left(\frac{\sqrt{5}}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = 1$$

$$\frac{5}{9} + \frac{4}{9} = 1$$

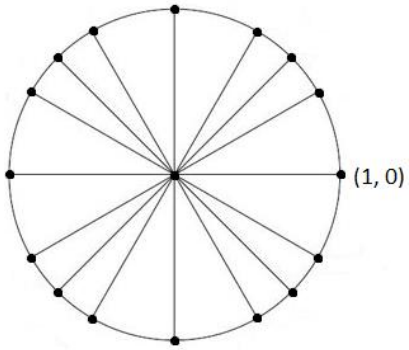
$$\frac{9}{9} = 1$$

Find the terminal point $P(x, y)$ on the unit circle determined by the given value of t.

$$t = 4\pi$$

Solution

Complete two full rotations in the counterclockwise direction, because $4\pi = 2\pi + 2\pi$.



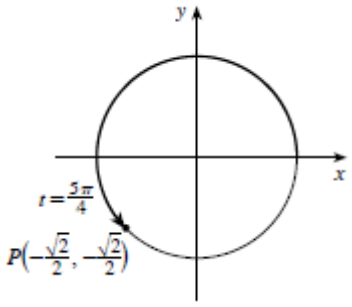
The terminal point will be $(1, 0)$.

Find the terminal point $P(x, y)$ on the unit circle determined by the given value of t.

$$t = \frac{5\pi}{4}$$

Solution

Complete half of a rotation, plus another 1/8 of a rotation (45 degrees) in the counterclockwise direction, because $\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \pi + \frac{\pi}{4}$.

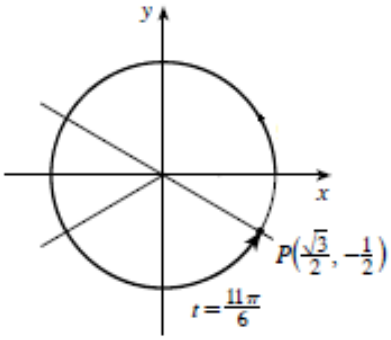


The terminal point will be $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Find the terminal point $P(x, y)$ on the unit circle determined by the given value of t.

$$t = \frac{11\pi}{6}$$

Solution



The terminal point will be $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

Find the reference number for each value of t.

a) $t = \frac{4\pi}{3}$

b) $t = \frac{5\pi}{3}$

c) $t = -\frac{7\pi}{6}$

d) $t = 3.5$

Solution

a) $\bar{t} = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

b) $\bar{t} = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

c) $\bar{t} = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$

d) $\bar{t} = 3.5 - \pi \approx 0.36$

Find a) the reference number for each value of t and b) the terminal point determined by t.

$$t = \frac{11\pi}{6}$$

Solution

a) $\bar{t} = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$

b) $P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Find a) the reference number for each value of t and b) the terminal point determined by t.

$$t = -\frac{2\pi}{6}$$

Solution

a) $\bar{t} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

b) $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Find a) the reference number for each value of t and b) the terminal point determined by t.

$$t = \frac{41\pi}{6}$$

Solution

a) $\bar{t} = 7\pi - \frac{41\pi}{6} = \frac{\pi}{6}$

b) $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$