Week 10

Sections 4.3, 4.4, 4.5

HW3: 10, 12, 18, 22, 26, 32, 46 (p. 351-352)
10, 12, 14, 36, 44, 52, 62 (p. 359)
16, 28, 50, 54, 58, 68 (p. 368)

Review Exercises
Express the equation in exponential form. a) $\log_3 \frac{1}{3} = -1$ b) $\log_4 16 = 2$ <u>Solution</u> a) $3^{-1} = \frac{1}{3}$ b) $4^2 = 16$
Express the equation in exponential form. $a)$ $\log_4 \frac{1}{64} = -3$ $b)$ $\log_9 3 = \frac{1}{2}$ <u>Solution</u> c) $4^{-3} = \frac{1}{64}$ d) $9^{\frac{1}{2}} = 3$
Express the equation in logarithmic form. a) $5^2 = 25$ b) $10^{-2} = \frac{1}{100}$ <u>Solution</u> a) $\log_5 25 = 2$ b) $\log_{10} \frac{1}{100} = -2$

Express the equation in logarithmic form. a) $4^{5x} = 11$ b) $7^{-5x} = 0.001$ <u>Solution</u> a) $\log_4 11 = 5x$ b) $\log_7 0.001 = -5x$
Evaluate the expression. a) $\log_6 6^{11}$ b) $\log_2 16$ c) $\log_3 9$ <u>Solution</u> a) $\log_6 6^{11} = 11$ b) $\log_2 16 = 4$ c) $\log_3 9 = 2$
Evaluate the expression. a) $\log_8 64$ b) $\log_3 3^5$ c) $\log_9 1$ <u>Solution</u> a) $\log_8 64 = 2$ b) $\log_3 3^5 = 5$ c) $\log_9 1 = 0$
Evaluate the expression. a) $e^{\ln\sqrt{5}}$ b) $e^{\ln\frac{2}{\pi}} = \frac{2}{\pi}$ c) $10^{\log 7}$ <u>Solution</u> a) $e^{ln\sqrt{5}} = \sqrt{5}$ b) $e^{\ln\frac{2}{\pi}} = \frac{2}{\pi}$ c) $10^{\log 7} = 7$

Use the calculator to evaluate the expression, correct to four decimal places.

a) $\log 34$
b) $\log \sqrt{8}$
c) $\log 6\sqrt{11}$

Solution

a) $\log 34 = 1.5314789 \ldots \approx 1.5315$
b) $\log \sqrt{8} = 0.4515449 \ldots \approx 0.4515$
c) $\log 6\sqrt{11} = 1.2988475 \ldots \approx 1.2988$

Use the Laws of Logarithms to evaluate the expression.

$$\log_4 8 + \log_4 2$$

Solution

$$\log_4 8 + \log_4 2$$

$$= \log_4(8 \cdot 2)$$

$$= \log_4 16$$

$$= 2$$

Use the Laws of Logarithms to evaluate the expression.

$$\log_2 60 - \log_2 15$$

Solution

$$\log_2 60 - \log_2 15$$

$$= \log_2 \left(\frac{60}{15}\right)$$

$$= \log_2 4$$

$$= 2$$

Use the Laws of Logarithms to evaluate the expression.

$$-\frac{1}{3}\log_5 125$$

Solution

$$-\frac{1}{3}\log_5 125 = -\frac{1}{3} \cdot 3 = -1$$

Use the Laws of Logarithms to expand the expression.

$$\log_7 \frac{x^5}{3y^2}$$

Solution

$$\log_7 \frac{x^5}{3y^2}$$

$$= \log_7 x^5 - \log_7 3 - \log_7 y^2$$

$$= 5\log_7 x - \log_7 3 - 2\log_7 y$$

Use the Laws of Logarithms to expand the expression.

$$\log_7 \frac{6x^4}{(x+1)^{11}}$$

Solution

$$\log_7 \frac{6x^4}{(x+1)^{11}}$$

$$= \log_7 6 + \log_7 x^4 - \log_7 (x+1)^{11}$$

$$= \log_7 6 + 4\log_7 x - 11\log_7 (x+1)$$

Use the Laws of Logarithms to combine the expression.

$$5\ln 3 + 6\ln x - \frac{1}{2}\ln(x-9)$$

Solution

$$5\ln 3 + 6\ln x - \frac{1}{2}\ln(x-9)$$

$$= \ln 3^5 + \ln x^6 - \ln\sqrt{x-9}$$

$$= \ln \frac{3^5 x^6}{\sqrt{x-9}}$$

$$= \ln \frac{243x^6}{\sqrt{x-9}}$$

Use the Change of Base Formula and a calculator to evaluate the logarithm, rounded to six decimal places. Use either natural or common logarithms.

$$\log_5 45$$

Solution

$$\log_5 45 = \frac{\log 45}{\log 5} = 2.365212389 \ldots \approx 2.365212$$

a. Find the exact solution of the exponential equation in terms of logarithms.
b. Use the calculator to find an approximation to the solution rounded to six decimal places.

$$2^{3x+1} = 7$$

Solution

a.

$$2^{3x+1} = 7$$

$$\ln 2^{3x+1} = \ln 7$$

$$(3x+1)\ln 2 = \ln 7$$

$$\frac{(3x+1)\ln 2}{\ln 2} = \frac{\ln 7}{\ln 2}$$

$$3x + 1 = \frac{\ln 7}{\ln 2}$$

$$-1 \quad -1$$

$$3x = \frac{\ln 7}{\ln 2} - 1$$

$$\frac{3x}{3} = \frac{\frac{\ln 7}{\ln 2} - 1}{3}$$

$$x = \frac{\frac{\ln 7}{\ln 2} - 1}{3}$$

b.

$$x = \frac{\frac{\ln 7}{\ln 2} - 1}{3} = 0.6024516407 \ldots \approx 0.602452$$

a. Find the exact solution of the exponential equation in terms of logarithms.
b. Use the calculator to find an approximation to the solution rounded to six decimal places.

$$4(8+2^{x+3})=84$$

Solution

a.

$$\frac{4(8+2^{x+3})}{4}=\frac{84}{4}$$

$$\begin{aligned}8+2^{x+3}&=21\\ -8\qquad &\ -8\end{aligned}$$

$$2^{x+3}=13$$

$$\ln 2^{x+3}=\ln 13$$

$$(x+3)\ln 2=\ln 13$$

$$\frac{(x+3)\ln 2}{\ln 2}=\frac{\ln 13}{\ln 2}$$

$$x+3=\frac{\ln 13}{\ln 2}$$

$$x=\frac{\ln 13}{\ln 2}-3$$

b.

$$x=\frac{\ln 13}{\ln 2}-3\approx 0.700440$$

Solve the logarithmic equation or x.

$$\log_8 x+\log_8(x+1)=\log_8 12$$

Solution

$$\log_8 x+\log_8(x+1)=\log_8 12$$
$$\log_8 x(x+1)=\log_8 12$$
$$x(x+1)=12$$
$$x^2+x=12$$
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x+4=0 \quad or \quad x-3=0$$
$$x=-4 \quad or \quad x=3$$

−4 is not a solution because logarithms of negative numbers are undefined.

The solution is $\{3\}$.

Solve the logarithmic equation or x.

$$\log_5 4 + \log_5(x+3) = \log_5 2 + \log_5(5x-1)$$

Solution

$$\log_5 4 + \log_5(x+3) = \log_5 2 + \log_5(5x-1)$$

$$\log_5 4(x+3) = \log_5 2(5x-1)$$

$$4(x+3) = 2(5x-1)$$

$$4x + 12 = 10x - 2$$

$$12 + 2 = 10x - 4x$$

$$14 = 6x$$

$$\frac{14}{6} = \frac{6x}{6}$$

$$x = \frac{7}{3}$$

The solution is $\left\{\frac{7}{3}\right\}$.

Solve the logarithmic equation or x.

$$\log(x-7) = 2$$

Solution

$$\log(x-7) = 2$$

$$x - 7 = 10^2$$

$$\begin{aligned} x - 7 &= 100 \\ +7 \quad &\; +7 \end{aligned}$$

$$x = 107$$

Solve the logarithmic equation or x.

$$\log_5(x+1) - \log_5(x-4) = 3$$

Solution

$$\log_5(x+1) - \log_5(x-4) = 3$$

$$\log_5 \frac{x+1}{x-4} = 3$$

$$\frac{x+1}{x-4} = 5^3$$

$$\frac{x+1}{x-4} = \frac{125}{1}$$

Use cross multiplying.

$$x + 1 = 125(x-4)$$

$$x + 1 = 125x - 500$$

$$1 + 500 = 125x - x$$

$$501 = 124x$$

$$\frac{501}{124} = \frac{124x}{124}$$

$$x = \frac{501}{124}$$