Learning Plan 07

Question 1

Hours of Sleep	Number of Citizens, in millions
4 or less	15
5	21
6	77
7	91
8	75
9	12
10 or more	9

Total: 300

What is the probability of a citizen getting at least 6 hours of sleep a night?

Solution

$$Probability = \frac{77 + 91 + 75 + 12 + 9}{300}$$

$$= \frac{264}{300}$$

$$= 0.88$$

$$= 88\%$$

Question 2

A 12-sided die is rolled.
The set of equally likely outcomes is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.
Find the probability of rolling a 3.

Solution

So, the die has 12 sides, but only one of those 12 sides has a “3” on it.
So, the probability is:

$$\frac{1}{12}$$

Question 3

A fair coin is tossed two times in succession. The set of equally likely outcomes is:
{HH, HT, TH, TT}.
Find the probability of getting a head on the first toss.

Solution

We have 4 outcomes: HH, HT, TH, TT
Do you notice that only two of them start with an H?

So, the probability is:

$$\frac{2}{4} = \frac{1}{2}$$

Question 4

		Second Roll					
		⚀	⚁	⚂	⚃	⚄	⚅
First Roll	⚀	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	⚁	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	⚂	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	⚃	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	⚄	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	⚅	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

A single die is rolled twice. The 36-equally likely outcomes are shown above. Find the probability of getting two numbers whose sum is 5.

Solution

Notice the pairs of numbers whose sum is 5:

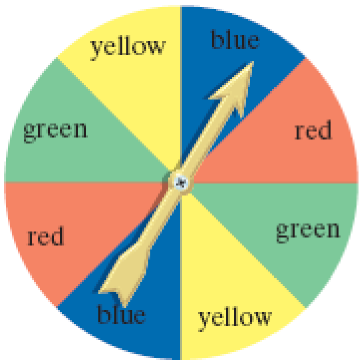
(1, 4)
(2, 3)
(3, 2)
(4, 1)

So, there are four pairs whose sum is 5.

The probability is:

$$\frac{4}{36} = \frac{1}{9}$$

Question 5



Assume that it is equally probable that the pointer will land on any one of the colored regions. If the pointer lands on a borderline, spin again.
If the spinner is spun once, find the probability that the pointer lands in a region that is red or blue.

Solution

The circle is divided into 8 sectors. Two of them are red, and two of them are blue.

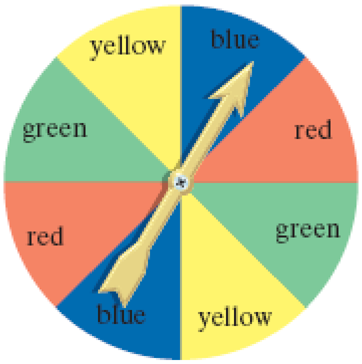
The probability is:

$$\frac{2}{8}+\frac{2}{8}=\frac{1}{4}+\frac{1}{4}$$

$$=\frac{2}{4}$$

$$=\frac{1}{2}$$

Question 6



Assume that it is equally probable that the pointer will land on any one of the colored regions. If the pointer lands on a borderline, spin again.
If the spinner is spun once, find the probability that the pointer lands in a region that is red and yellow.

Solution

Pay attention to the word "and". Red and Yellow

If you spin only once, you can't get red and yellow in the same time, so the probability is 0.

Question 7

Sickle cell anemia is an inherited disease in which red blood cells become distorted and deprived of oxygen. A person with two sickle cell genes will have the disease, but a person with only one sickle cell gene will have a mild, nonfatal anemia called sickle cell trait. If we use Upper S to represent a healthy gene, and s a sickle cell gene, the table shows the four possibilities for the children of one healthy, SS parent, and one parent with sickle cell trait, Ss.

Find the probability that these parents give birth to a child who has sickle cell anemia.

		Second Parent (with Sickle Cell Trait) S	s
Healthy First Parent	S	SS	Ss
	S	SS	Ss

Solution

SS – Healthy Person
Ss – Person with a Sickle Cell Trait
ss – Person with a Sickle Cell Anemia

We need to find the probability of getting a child who has sickle cell anemia (ss).

The table shows four outcomes: SS, SS, Ss, Ss. None of these outcomes are ss.

So, the probability is:

$$\frac{0}{4} = 0$$

Question 8

This problem involves empirical probability. The table shows the breakdown od 102 thousand single parents in active duty in the U.S. military in a certain year. All numbers are in thousands and rounded to the nearest thousand. Use the data in the table to find the probability that a randomly selected single parent in the U.S. military is a woman in the Air Force. (Round to the nearest hundredth)

	Army	Navy	Marine Corps	Air Force	Total
Male	27	27	7	15	76
Female	11	8	1	6	26
Total	38	35	8	21	102

Solution

We have a total of 102 thousand single parents. From the table, there are 6 thousand women in Air Force.

$$\frac{6}{102} = 0.0588\ldots$$

$$\approx 0.06$$

Question 9

The table shows the educational attainment of a country's population, aged 25 and over. Use the data in the table, expressed in millions, to find the probability that a randomly selected citizen, aged 25 or over, had 4 years of college. (Type an integer or a fraction).

	Less Than 4 Years High School	4 Years High School Only	Some College (Less Than 4 Years)	4 Years College (or More)	Total
Male	13	23	20	25	81
Female	14	30	24	20	88
Total	27	53	44	45	169

Solution

$$\frac{45}{169}$$

Question 10

If you are dealt 3 cards from a shuffled deck of 52 cards, find the probability that all three cards are queens.

Solution

There are 4 queens in a desk of cards.
The probability that you are dealt a queen is ${}^{4}/_{52}$.

You keep the card, so now you are left with only 51 cards, and only 3 of them being queens.

So, the probability of getting another queen is ${}^{3}/_{51}$.

You keep the second card, so now you are left with only 50 cards, and only 2 of them being queens.

So, the probability of getting another queen is ${}^{2}/_{50}$.

The probability that all three cards are queens is:

$$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = 0.000181$$

This is how you should put this on your calculator:

$$4 \div 52 \cdot 3 \div 51 \cdot 2 \div 50 = 0.000181$$

Question 11

You randomly select one card from a shuffled deck of 52 cards. Find the probability of selecting a black nine or a black five.

Solution

There are 2 black nines and 2 black fives in a desk of cards.
So, the probability is:

$$\frac{2}{52} + \frac{2}{52} = \frac{4}{54} = \frac{1}{13}$$

Question 12

A group consists of 6 men and 5 women. Four people are selected to attend a conference. In how many ways can 4 people be selected from a group of 11 people? In how many ways can 4 men be selected from 6 men? Find the probability that the selected group will consist of all men.

Solution

Because the order does not matter, we will use combinations.

$$11C4 = \frac{11!}{(11-4)!\,4!}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!\,4!}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \text{330 ways}$$

$$6C4 = \frac{6!}{(6-4)!\,4!}$$

$$= \frac{6 \cdot 5 \cdot 4!}{2!\,4!}$$

$$= \frac{6 \cdot 5}{2}$$

$$= \text{15 ways}$$

$$P(all\ men) = \frac{number\ of\ ways\ to\ select\ 4\ men}{total\ number\ of\ possible\ combinations}$$

$$= \frac{15}{330}$$

$$= \frac{15 \div 15}{330 \div 15}$$

$$= \frac{1}{22}$$

Question 13

To play the lottery in a certain state, a person has to correctly select 5 out of 45 numbers, playing $1 for each five-number selection. If the five numbers picked are the same as the drawn by the lottery, an enormous sum of money is bestowed. What is the probability that a person with one combination of five numbers will win? What is the probability of winning if 100 different lottery tickets are purchased?

Solution

The order in which you select the five numbers does not matter. Let's find in how many ways you can choose 5 out of 45 numbers.

$$45C5 = \frac{45!}{(45-5)!\,5!}$$

$$= \frac{45 \cdot 44 \cdot 43 \cdot 42 \cdot 41 \cdot 40!}{40! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{45 \cdot 44 \cdot 43 \cdot 42 \cdot 41}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 1{,}221{,}759 \; ways$$

The probability that a person with one combination of five numbers will win is:

$$\frac{1}{1{,}221{,}759}$$

The probability of winning if 100 different lottery tickets are purchased is:

$$\frac{100}{1{,}221{,}759}$$

Question 14

A committee consisting of 6 people is to be selected from eight parents and four teachers. Find the probability of selecting three parents and three teachers.

Solution

The order in which you select the 6 people does not matter. Let's find in how many ways you can choose 6 out of 12 people (8 parents + 4 teachers).

$$12C6 = \frac{12!}{(12-6)!\,6!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 924\ ways$$

Now let's find in how many ways you can select three parents and three teachers.

$$8C3 \cdot 4C3 = \frac{8!}{(8-3)!\,3!} \cdot \frac{4!}{(4-3)!\,3!}$$

$$= 224\ ways$$

The probability of selecting three parents and three teachers is:

$$\frac{224}{924} = \frac{224 \div 28}{924 \div 28} = \frac{8}{33}$$

You can do this entire problem on your calculator by typing:

$$8C3 \cdot 4C3/12C6 = 8/33$$

Question 15

You are dealt one card from a 52-card deck. Find the probability that you are not dealt a heart.

Solution

The are 13 hearts in a deck of cards.
From 1 (1 represents 100% of all the cards, or the entire deck), subtract the probalility of selecting a heart.

$$1-\frac{13}{52}$$

$$=\frac{52}{52}-\frac{13}{52}$$

$$=\frac{39}{52}$$

$$=\frac{39 \div 13}{52 \div 13}$$

$$=\frac{3}{4}$$

Question 16

A single die is rolled twice. The set of 36 equality outcomes is {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3, (6,4), (6,5), (6,6)}.
Find the probability of getting a sum of 6 or 7.

Solution

		Second Roll					
		⚀	⚁	⚂	⚃	⚄	⚅
First Roll	⚀	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	⚁	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	⚂	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	⚃	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	⚄	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	⚅	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Find the diagonal that will give you the sum of 6:

(1, 5)
(2, 4)
(3, 3)
(4, 2)
(5, 1)

So, there are 5 pairs whose sum is 6.

Now find the diagonal that will give you the sum of 7:

(1, 6)
(2, 5)
(3, 4)
(4, 3)
(5, 2)
(6, 1)

So, there are 6 pairs whose sum is 6.

The probability is:

$$\frac{5}{36} + \frac{6}{36} = \frac{11}{36}$$

Question 17

The physics department of a college has 5 male professors, 12 female professors, 10 male teaching assistants, and 8 female teaching assistants. If a person is selected at random from a group, find the probability that the selected person is a teaching assistant or a female.

Solution

Find the total amount of people:

$$5 + 12 + 10 + 8 = 35$$

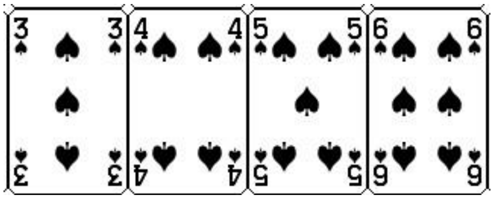
Now add the probabilities of 10 male teaching assistants, and 8 female teaching assistants, and 12 female professors.

$$\frac{10}{35} + \frac{8}{35} + \frac{12}{35} = \frac{30}{35} = \frac{6}{7}$$

Question 18

One card is randomly selected from a deck of cards. Find the odds against drawing a spade greater than 2 and less than 7.

Solution



In a desk of 52 cards, we have 4 cards that spades greater than 2 and less than 7.

How many are not spades greater than 2 and less than 7?

$$52 - 4 = 48$$

The odds against drawing a spade greater than 2 and less than 7 is:

$$48 : 4 = 12 : 1$$

Question 19

A spinner is used for which it is equally probable that the pointer will land on any one of six regions. Three of the regions are colored red, two are colored green, and one is colored yellow. If the pointer is spun three times, find the probability it will land on green every time.

Solution

There are two regions colored in green. The probability of getting a "green" each individual time is 2/6.
To get "green" and "green" and again "green" three times in a row, you will multiply their probabilities.

$$\frac{2}{6}\cdot\frac{2}{6}\cdot\frac{2}{6}=\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{27}$$

Question 20

A single die is rolled twice. Find the probability of getting a 5 the first time and a 1 the second time.

Solution

The probability of getting a 5 is 1/6.
The probability of getting a 1 is also 1/6.
To find the probability of getting a 5 the first time and a 1 the second time, mutiply:

$$\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$$

Question 21

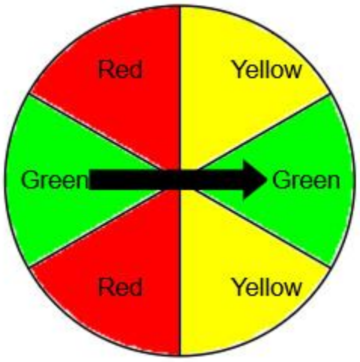
You are dealt one card from a 52-card deck. Then the card is replaced in the deck, the deck is shuffled, and you draw again. Find the probability of getting a picture card the first time and a heart the second time.

Solution

There are 12 pictures cards and 13 hearts. To find the probability of getting a picture card the first time and a heart the second time, multiply the probabilites.

$$\frac{12}{52}\cdot\frac{13}{52}=\frac{12\div 4}{52\div 4}\cdot\frac{13\div 13}{52\div 13}=\frac{3}{13}\cdot\frac{1}{4}=\frac{3}{52}$$

Question 22



If the pointer is spun twice, find the probability that it will land on yellow and then on yellow.

Solution

The circle is divided into 6 regions and 2 of them are yellow.

$$\frac{2}{6} \cdot \frac{2}{6} = \frac{4}{36} = \frac{1}{9}$$

Question 23

Find the probability that the spinner will land on purple and then green and then blue.

Solution

The circle is divided into 8 regions.
1 region is purple, 2 regions are green, and 3 regions are blue.

$$\frac{1}{8} \cdot \frac{2}{8} \cdot \frac{3}{8} = \frac{3}{256}$$

Question 24

A coin is tossed and a die is rolled. Find the probability of getting a head and a number greater than 5.

Solution

The probability of getting a head is $^1/_{2.}$

The probability of getting a number greater than 5 is $^1/_{6.}$ (because only 6 is greater than 5)

$$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$