Learning Plan 06

Chapter 11

Question 1

A sales representative can take one of 3 different routes from City C to City F and any one of 5 different routes from City F to city M. How many different routes can she take from City C to City M going through City F?

Solution

$$3 \cdot 5 = 15\ ways$$

Question 2

A restaurant offers the following limited lunch menu.

Main Courses: Turkey, Spaghetti, Meatloaf, Shrimp (4 items)
Vegetables: Corn, Green Beans (2 items)
Beverages: Coffee, Tea, Milk (3 items)
Desserts: Sundaes, Mousse (2 items)
If one item is selected from each of the four groups, in how many ways can a meal be ordered?

Solution

$$4 \cdot 2 \cdot 3 \cdot 2 = 48\ ways$$

Question 3

License plates in a particular state display 3 letters followed by 2 numbers. How many different license plates can be manufactured for this state?

Solution

There are 26 letters in the English alphabet.
There are 10 one-digit numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

You need 3 letters and then 2 numbers.

Use the Counting Principle:

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 = 1{,}757{,}600$$ license plates.

Question 4

How many three-digit even numbers are possible if the leftmost digit cannot be zero?

Solution

First, here are some examples of three-digit even numbers: 326, 508, 122, 880…

$$9 \cdot 10 \cdot 5 = 450 \text{ numbers}$$

The first number is 9, because in the first position you can have one of the following nine non-zero numbers: $1, 2, 3, 4, 5, 6, 7, 8, 9$	The second number is 10, because in the second position you can have one of the following ten numbers: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$	The third number is 5, because in the third position you can have one of the following five even numbers: $0, 2, 4, 6, 8$

If you are asked for six-digit odd numbers, you will need: $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 5$

Question 5

You are taking an online survey. There are 10 questions with each question having 4 choices. In how many ways can you answer the questions?

Solution

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{10} = 1{,}048{,}576 \text{ ways.}$$

Question 6

Find the definition in the textbook.

Question 7

You need to arrange ten of your favorite books along a small shelf. How many different ways can you arrange the books, assuming that the order of the books makes a difference to you?

Solution

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880 \text{ ways.}$$

You can also use your calculator to type $10!$, and you will get the same answer.

Question 8

Evaluate:

$$\frac{29!}{26!}$$

Solution

$$\frac{29!}{26!} = \frac{29 \cdot 28 \cdot 27 \cdot 26!}{26!}$$

$$= 29 \cdot 28 \cdot 27$$

$$= 21{,}924$$

Also, try to do this problem on you calculator. First find the factorial ! symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "!". Then type:

$$29! \div 26!$$

Permutations	Combinations
An arrangement of objects in which **the order matters.**	An arrangement of objects in which **the order doesn't matter.**
$\mathrm{nPr} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!}$	$\mathrm{nCr} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!\,\mathrm{r}!}$
Example A club with 10 members has to choose three officers - president, vice-president, and treasurer. In how many ways can these positions be filled? (The order matters, because each person is assigned a certain position, so this is a permutation problem) $\mathrm{nPr} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!}$	Example A club with 10 members has to choose a three-person committee. How many different committees are possible? (The order does not matter, because three chosen people are not assigned a certain position, or title, so this is a combination problem) $\mathrm{nCr} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!\,\mathrm{r}!}$

$= \frac{10!}{(10-3)!}$ $= \frac{10!}{7!}$ $= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$ $= 10 \cdot 9 \cdot 8$ $= 720$ ways.	$= \frac{10!}{(10-3)!\,3!}$ $= \frac{10!}{7!\,3!}$ $= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1}$ $= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$ $= 120$ ways.

Questions 9, 10

A club with ten members is to choose three officers – president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?

Solution

$$\mathrm{nPr} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720\ ways$$

Also, try to do this problem on you calculator. First find the Permutation symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "nPr". Then type:

$$10\mathrm{P}3$$

$$= 720$$

Questions 11 and 12

In how many distinct ways can the letters of the word APPEAR be arranged?

Solution

The word APPEAR has 6 letters, but A repeats twice, and P repeats twice. Therefore, you will take 6! and divide by the factorial of the number of times these letters repeat. In the textbook, the formula is on page 699.

$$\frac{6!}{2!\,2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 180\ ways$$

Questions 13, 14, 15

An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

Solution

$$\mathrm{nCr} = \frac{\mathrm{n!}}{(\mathrm{n}-\mathrm{r})!\,\mathrm{r!}} = \frac{6!}{(6-3)!\,3!} = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3 \cdot 2 \cdot 1} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20\ ways$$

Also, try to do this problem on you calculator. First find the Combination symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "nCr". Then type:

$$6\mathrm{C}3$$

$$= 20$$

Question 16

The senate in a certain state is comprised of 52 Republicans, 45 Democrats, and 3 Independents. How many committees can be formed if each committee must have 3 Republicans and 2 Democrats?

Solution

First of all, we will not use the Independents.

Now, just type on the calculator:

$$52C3 \cdot 45C2 = 21{,}879{,}000 \text{ committees.}$$

Questions 17, 18

A club with ten members is to choose three officers – president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?

Solution

$$\mathrm{nPr} = \frac{\mathrm{n!}}{(\mathrm{n}-\mathrm{r})!} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720\ ways$$

Also, try to do this problem on you calculator. First find the Permutation symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "nPr". Then type:

$$10\mathrm{P}3 = 720$$

Questions 19, 20

Solve the problem by applying the Counting Principle.
A restaurant offers a choice of 4 salads, 5 main courses, and 3 desserts. How many possible 3-course meals are there?

Solution

$$4 \cdot 5 \cdot 3 = 60\ meals$$

Questions 21

Solve the problem by applying the Counting Principle.
There are 5 performers who are to present their acts at a variety show. One of them insists on being the first act of the evening. If the request is granted, how many different ways are there to schedule the appearances?

Solution

There is only one way you can select the first person, so you will have number 1 in the first position. Once the first person is selected, you have only 4 people to choose from in the second position, then 3 people to choose from in the third position, 2 people in the forth position, and 1 person in the fifth position.

$$1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24\ ways$$

Question 22

Evaluate:

$$\frac{300!}{299!}$$

Solution

$$\frac{300!}{299!} = \frac{300 \cdot 299!}{299!}$$

$$= 300$$

Also, try to do this problem on you calculator. First find the factorial ! symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "!". Then type:

$$300! \div 299!$$

Question 23

Evaluate:

$$(7-3)!$$

Solution

$$(7-3)!$$

$$= 4!$$

$$= 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 24$$

Also, try to do this problem on you calculator. First find the factorial ! symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2nd" or "Shift" key to access "!". Then type:

$$(7-3)!$$

Questions 24, 25

A club with ten members is to choose three officers – president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?

Solution

$$\mathrm{nPr} = \frac{\mathrm{n!}}{(\mathrm{n}-\mathrm{r})!} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720\ ways$$

Also, try to do this problem on you calculator. First find the Permutation symbol. Consult the internet for info on your specific calculator. On many calculators, you have to press "2^{nd}" or "Shift" key to access "nPr". Then type:

$$10\mathrm{P}3 = 720$$