

Exponential Functions

Definition of an Exponential Function

An exponential function with base b is defined by $f(x) = b^x$, where $b > 0$, $b \neq 1$, and x is any real number.

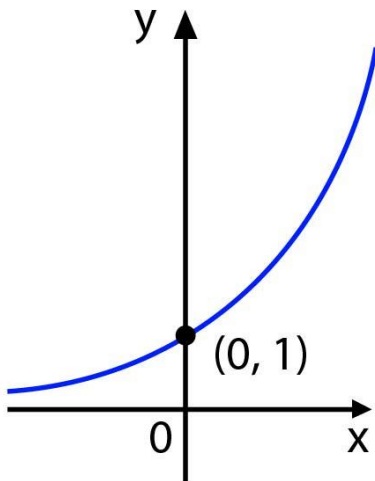
Examples:

$$f(x) = 4^x, \quad g(x) = 10^{5x}, \quad h(x) = -\left(\frac{2}{3}\right)^{x-2}$$

Note, that in exponential functions, x is part of the exponent.

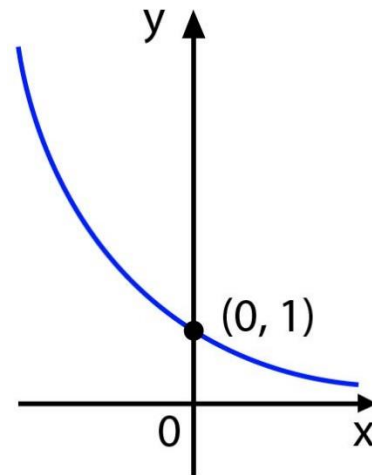
Graphs of Exponential Functions

$$f(x) = a^x \text{ for } a > 1$$



The domain is all the real numbers and the range is $(0, \infty)$.
The line $y = 0$ (x-axis) is a horizontal asymptote of f .

$$f(x) = a^x \text{ for } 0 < a < 1$$



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Number e

Number e is an irrational number and the approximate value is $e \approx 2.718281827 \dots$

The number e is called **natural base** and is defined as the value that $\left(1 + \frac{1}{n}\right)^n$ approaches as n gets larger and larger.

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

The number e was named by the Swiss mathematician **Leonard Euler** (1707-1783).

Example of Graphing an Exponential Function

Graph the exponential function $y = 3^x$.

Solution

Construct a table with values for x and y . Then plot the points to sketch the graph.

x	y
-2	$y = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
-1	$y = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$
0	$y = 3^0 = 1$
1	$y = 3^1 = 3$
2	$y = 3^2 = 9$

