

Solving Linear Equations

Definition of a Linear Equation

A linear equation in one variable is an equation that can be written in the form

$$ax + b = 0, \text{ where } a \text{ and } b \text{ are real numbers and } a \neq 0.$$

Addition Property of Equality		Multiplication Property of Equality	
If $a, b,$ and c are real numbers, then $a = b$ is equivalent to $a + c = b + c$	If $a, b,$ and c are real numbers, then $a = b$ is equivalent to $a - c = b - c$	If $a, b,$ and c are real numbers, and $c \neq 0$ then $a = b$ is equivalent to $a \cdot c = b \cdot c$	If $a, b,$ and c are real numbers, and $c \neq 0$ then $a = b$ is equivalent to $\frac{a}{c} = \frac{b}{c}$
Example: $x - 7 = 3$ $+7 \quad +7$ $x = 10$	Example: $x + 5 = -2$ $-5 \quad -5$ $x = -7$	Example: $\frac{x}{7} = 4$ $\frac{x}{7} \cdot 7 = 4 \cdot 7$ $x = 28$	Example: $6x = -12$ $\frac{6x}{6} = \frac{-12}{6}$ $x = -2$

Examples of Solving Linear Equations:

Examples:	Explanation:
<p>Solve the linear equation:</p> $8x + 3 = 5x - 7$ <p><u>Solution</u></p> $\begin{array}{r} 8x + 3 = 5x - 7 \\ -3 \quad -3 \end{array}$ $\begin{array}{r} 8x = 5x - 10 \\ -5x \quad -5x \end{array}$ $3x = -10$ $\frac{3x}{3} = \frac{-10}{3}$ $x = -\frac{10}{3}$	<p>Subtract 3 from both sides.</p> <p>Subtract $5x$ from both sides.</p> <p>Divide both sides by 3.</p>
<p>Solve the linear equation:</p> $6 - 4x = 3(5x - 1) - (x + 9)$ <p><u>Solution</u></p> $6 - 4x = 3(5x - 1) - (x + 9)$ $6 - 4x = 15x - 3 - x - 9$ $\begin{array}{r} 6 - 4x = 14x - 12 \\ +12 \quad +12 \end{array}$ $\begin{array}{r} 18 - 4x = 14x \\ +4x \quad +4x \end{array}$ $18 = 18x$ $\frac{18}{18} = \frac{18x}{18}$ $x = 1$	<p>Use distributive property to remove the parentheses on the right side.</p> <p>Combine like terms on the right side.</p> <p>To isolate x on the right side, add 12 to both sides, then add $4x$ to both sides.</p> <p>Divide both sides by 18.</p>

Solve the linear equation:

$$\frac{x}{3} - \frac{1}{5} = \frac{x}{2}$$

Solution

$$LCD = 30$$

$$30\left(\frac{x}{3} - \frac{1}{5}\right) = \left(\frac{x}{2}\right)30$$

$$\begin{array}{r} 10x - 6 = 15x \\ -10x \quad -10x \end{array}$$

$$-6 = 5x$$

$$-\frac{6}{5} = \frac{5x}{5}$$

$$x = -\frac{6}{5}$$

To eliminate the fractions, we need to multiply both sides by the Least Common Denominator (LCD).

The Least Common Denominator (LCD) for all three denominators **3, 5, and 2** is **30**. Multiply both sides by 30.

$$30 \cdot \frac{x}{3} = 10x \quad 30 \cdot \left(-\frac{1}{5}\right) = -6 \quad 30 \cdot \frac{x}{2} = 15x$$

To isolate x on the right side, subtract $10x$ from both sides.

Divide both sides by 5.

Solve the linear equation:

$$\frac{x}{4} - 7 = \frac{x-5}{2}$$

Solution

$$LCD = 4$$

$$4\left(\frac{x}{4} - 7\right) = \left(\frac{x-5}{2}\right)4$$

$$\begin{array}{r} x - 28 = 2x - 10 \\ -x \quad -x \end{array}$$

$$-28 = x - 10$$

$$+10 \quad +10$$

$$-18 = x$$

To eliminate the fractions, we need to multiply both sides by the Least Common Denominator (LCD).

The Least Common Denominator (LCD) for both denominators **4 and 2** is **4**. Multiply both sides by 4.

$$4 \cdot \frac{x}{4} = x \quad 4 \cdot (-7) = -28$$

$$4 \cdot \frac{x-5}{2} = 2(x-5) = 2x - 10$$

To isolate x on the right side, subtract x from both sides, then add 10 to both sides.