Types of Equations Based on the Solution Set

Identity	Conditional Equation	Inconsistent Equation
An Identity is an equation with indefinitely many solutions. The solutions can be any numbers for which both sides of the identity equation are defined.	A conditional equation has one or more solutions, but it is not an identity. A conditional equational has a finite number of solutions.	An inconsistent equation is an equation that has no solutions.

Example	Example	Example
$4(x + 3) = 4x + 12$ $$4x + 12 = 4x + 12$$ $$\begin{array}{r} -12 \quad\quad -12 \end{array}$$ $$4x = 4x$$ $$-4x \quad -4x$$ $$0 = 0$$ The solution is any real number R.	$x + 7 = 10$ $$\begin{array}{r} -7 \quad -7 \end{array}$$ $$x = 3$$ The equation has one solution $\{3\}$.	$x = x + 3$ $$-x \quad -x$$ $$0 \neq 3$$ The equation has no solutions, because no matter what number is used in place of x, the right side is always 3 more than the left side.

Another Example	Another Example	Another Example
$$\frac{x}{x} = 1$$ Any number (except zero) divided by the same number equals 1. So, the solution is any real number, except 0. $$(\infty, 0) \cup (0, \infty)$$	$$x^2 = 16$$ $$x = \pm\sqrt{16}$$ $$x = \pm 4$$ The equation has two solutions: $$\{-4, 4\}$$	$$\frac{5x}{x + 2} = 4 - \frac{10}{x + 2}$$ $$(x + 2)\left(\frac{5x}{x + 2}\right) = \left(4 - \frac{10}{x + 2}\right)(x + 2)$$ $$5x = 4(x + 2) - 10$$ $$5x = 4x + 8 - 10$$ $$5x - 4x = -2$$ $$x = -2$$ The proposed solution -2 is not a solutin to this equation, because it makes the denominator equal to 0. So, the solution is empty set $\emptyset$.