# Sets; Intersection and Union of Sets

## Definition of a Set

A set is a collection of objects.

The objects that belong to a set are called elements, or members of the given set.
We use uppercase letters for the names of sets.

One common method of describing a set, is the roster method.
With this method, we include the elements of a set inside the braces { }.

### Examples of Sets

$A = \{1, 5, 7, 8\}$ $B = \{a, c, f, h, k\}$ $N = \{1, 2, 3, 4, \dots\}$

| A set that has a finite number of elements is called a Finite Set *(we can count the elements)* | A set that has an infinite number of elements is called an Infinite Set *(we cannot count the elements)* |
|---|---|
| Examples<br>$B = \{4, 6, 11\}$<br>$D = \{1, 2, 3, 4, 5, 6\}$ | Examples<br>$N = \{1, 2, 3, 4, \dots\}$<br>$O = \{1, 3, 5, 7, \dots\}$ |

## The Symbol ∈

The symbol ∈ is used to indicate whether a certain object is an element of a set.

We read ∈ - "is an element of"
We read ∉ - "is not an element of"

### Example

Let $A = \{2, 4, 5, 7, 8\}$ be a set.

$2 \in A$ (we read it: "Two is an element of set A"). And it is true that 2 belongs to set A.
$3 \notin A$ (we read it: "Three is not an element of set A") And it is true that 3 does not belong to set A.

A set that contains no elements is an empty set and is represented by $\emptyset$ or $\{\ \}$.

## Three most Common Ways of Describing a Set

(using the elements $1, 2, 3, 4, 5$)

| Roster Method *(List all the elements inside the braces)* | Set-Builder Notation *(Use a variable $x$)* | Word Description *(use words to describe a set)* |
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| $A = \{1, 2, 3, 4, 5\}$ | $A = \{x \mid x \in N \text{ and } x < 6\}$ | $A$ is the set of the natural numbers less than 6. |

How to read it:

A is the set of all elements $x$, such that $x$ is an element of the natural numbers, and $x$ is less than 6.

## The Intersection ∩ of Two Sets

The intersection of sets $A$ and $B$, written $A \cap B$, is the set of elements common to both set $A$ and set $B$.

### Example

Two sets are given:

$A = \{2, 3, 6, 8)$ $\quad$ $B = \{1, 2, 3, 4, 5, \}$

The intersection of these two sets is:

$A \cap B = \{2, 3\}$

because the numbers 2 and 3 are common to both sets A and B.

## The Union ∪ of Two Sets

The union of sets $A$ and $B$, written $A \cup B$, is the set of elements that are members of set $A$ or of set $B$ or of both sets.

### Example

Two sets are given:

$A = \{2, 3, 6, 8)$ $\quad$ $B = \{1, 2, 3, 4, 5, \}$

The union of these two sets is:

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

because these numbers belong to either A or B or both.

In other words, we list all the numbers together, starting with the smallest to the largest. And if some of them repeat, list them only once.

# Exercises

| | |
|---|---|
| Given the set $A = \{1, 4, 7, 9\}$, determine whether the number $4$ is an element of this set.<br><br>Solution<br><br>Yes, $4$ is an element of set A.<br>$4 \in A$ | Given the set $A = \{1, 4, 7, 9\}$, determine whether the number $5$ is an element of this set.<br><br>Solution<br><br>No, $5$ is not an element of set A.<br>$5 \notin A$ |
| Two sets are given:<br>$A = \{2, 4, 5, 9, 10\}$ and $B = \{4, 5, 6\}$.<br>Determine the intersection ∩ of these two sets.<br><br>Solution<br><br>The numbers that belong to both A and B, (or the common numbers) are $4$ and $5$. So,<br><br>$A \cap B = \{4, 5\}$ | Two sets are given:<br>$A = \{2, 4, 5, 9, 10\}$ and $B = \{4, 5, 6\}$.<br>Determine the union ∪ of these two sets.<br><br>Solution<br><br>The numbers that belong to either A or B, or both sets, are all the given numbers written from the smallest to the largest. (If some of them repeat, write them only once). So,<br><br>$A \cup B = \{2, 4, 5, 6, 9, 10\}$ |
| Two sets are given:<br>$A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$.<br>Determine the intersection ∩ of these two sets.<br><br>Solution<br><br>There are no common numbers to both A and B, so the intersection is the empty set.<br><br>$A \cap B = \emptyset$ | Two sets are given:<br>$A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$.<br>Determine the union ∪ of these two sets.<br><br>Solution<br><br>The numbers that belong to either A or B, or both sets, are all the given numbers written from the smallest to the largest. So,<br><br>$A \cup B = \{1, 2, 3, 4, 5, 6\}$ |
| Two sets are given:<br>$A = \{7, 8, 11\}$ and $B = \{\ \}$.<br>Determine the intersection ∩ of these two sets.<br><br>Solution<br><br>Notice that set B is an empty set.<br>There are no common numbers to both A and B, so the intersection is the empty set.<br><br>$A \cap B = \{\ \}$ or $\emptyset$ | Two sets are given:<br>$A = \{7, 8, 11\}$ and $B = \{\ \}$.<br>Determine the union ∪ of these two sets.<br><br>Solution<br><br>The numbers that belong to either A or B, or both sets, are all the given numbers written from the smallest to the largest. So,<br><br>$A \cup B = \{7, 8, 11\}$ |