

The Wheat and the Chessboard Problem



The legend says that a peasant invented the game of chess and brought it as a gift to the king. The king liked the game so much, that he offered the peasant any gift he wanted in exchange.

The peasant told the king that all he wanted was **1 grain of wheat on the first square, 2 grains on the second square, 4 grains on the third square, and so on, each time doubling the amount grains until the last square of the chessboard.**

The king was surprised by the modest request, and called his servants to count the grains. But soon the king realized that the request was impossible to fulfil, because of the enormous amount of grain needed.

So how much wheat was needed?

There are 64 squares on the chessboard.

The expression 2^{n-1} (where n represents a certain square on the chessboard), gives us the number of grains on any given square.

For example: The number of grains on square 5 is $2^{5-1} = 2^4 = 16$ grains.

We will find the sum of the grains on all 64 squares using the sum symbol \sum .

$$\sum_{n=1}^{64} 2^{n-1} = 2^{1-1} + 2^{2-1} + 2^{3-1} \dots + 2^{64-1} = 18,446,744,073,709,600,000$$

**This is about 18 quintillion grains of wheat,
and it is about 1615 more than the current world production of wheat per year.**