

How to Convert a Repeating Decimal into a Fraction

Example 1

Convert the decimal 0.47474747 ... into a fraction.

1. Use a letter x to represent the decimal 0.47474747 ...

$$x = 0.4747474747 \dots \quad (\text{Let's call it Equation 1})$$

2. Multiply both sides of the equation by 100, because the decimal has 2 repeating digits. When multiplying the right side by 100, move the decimal point two places to the right.

$$100(x) = 100(0.4747474747 \dots)$$

$$100x = 47.4747474747 \dots \quad (\text{Let's call it Equation 2})$$

3. Subtract equation 1 from equation 2.

$$\begin{array}{r} 100x = 47.4747474747 \dots \\ - \quad x = 0.4747474747 \dots \\ \hline 99x = 47 \end{array}$$

4. Solve the equation for x .

$$99x = 47$$

$$\frac{99x}{99} = \frac{47}{99}$$

$$x = \frac{47}{99}$$

So, we converted the repeating decimal into a fraction.

$$0.47474747474747 \dots = \frac{47}{99}$$

Example 2

Convert the decimal $0.222222 \dots$ into a fraction.

1. Use a letter x to represent the decimal $0.222222 \dots$

$$x = 0.222222 \dots \quad (\text{Let's call it Equation 1})$$

2. Multiply both sides of the equation by 10, because the decimal has 1 repeating digit. When multiplying the right side by 10, move the decimal point one place to the right.

$$10(x) = 10(0.222222 \dots)$$

$$10x = 2.22222 \dots \quad (\text{Let's call it Equation 2})$$

3. Subtract equation 1 from equation 2.

$$\begin{array}{r} 10x = 2.22222 \dots \\ - \quad x = 0.222222 \dots \\ \hline 9x = 2 \end{array}$$

4. Solve the equation for x .

$$9x = 2$$

$$\frac{9x}{9} = \frac{2}{9}$$

$$x = \frac{2}{9}$$

So, we converted the repeating decimal into a fraction.

$$0.222222 \dots = \frac{2}{9}$$

Example 3

Convert the decimal 0.285714285714 ... into a fraction.

1. Use a letter x to represent the decimal 0.285714285714 ...

$$x = 0.285714285714 \dots \text{ (Let's call it Equation 1)}$$

2. Multiply both sides of the equation by 1,000,000, because the decimal has 6 repeating digits. When multiplying the right side by 1,000,000, move the decimal point six places to the right.

$$1,000,000(x) = 1,000,000(0.285714285714 \dots)$$

$$1,000,000x = 285,714.285714 \dots \text{ (Let's call it Equation 2)}$$

3. Subtract equation 1 from equation 2.

$$\begin{array}{r} 1,000,000x = 285,714.285714 \dots \\ - \quad \quad \quad x = 0.285714285714 \dots \\ \hline 999,999x = 285,714 \end{array}$$

4. Solve the equation for x .

$$999,999x = 285,714$$

$$\frac{999,999x}{999,999} = \frac{285,714}{999,999}$$

$$x = \frac{285,714}{999,999}$$

Reduce the fraction.

$$x = \frac{285,714 \div 142,857}{999,999 \div 142,857} = \frac{2}{7}$$

So, we converted the repeating decimal into a fraction.

$$0.285714285714 \dots = \frac{2}{7}$$

Example 4

Convert the decimal 0.45454545 ... into a fraction.

1. Use a letter x to represent the decimal 0.45454545 ...

$$x = 0.45454545 \dots \quad (\text{Let's call it Equation 1})$$

2. Multiply both sides of the equation by 100, because the decimal has 2 repeating digits. When multiplying the right side by 100, move the decimal point two places to the right.

$$100(x) = 100(0.45454545 \dots)$$

$$100x = 45.45454545 \dots \quad (\text{Let's call it Equation 2})$$

3. Subtract equation 1 from equation 2.

$$\begin{array}{r} 100x = 45.45454545 \dots \\ - \quad x = 0.45454545 \dots \\ \hline 99x = 45 \end{array}$$

4. Solve the equation for x .

$$99x = 45$$

$$\frac{99x}{99} = \frac{45}{99}$$

$$x = \frac{45}{99}$$

Reduce the fraction.

$$x = \frac{45 \div 9}{99 \div 9} = \frac{5}{11}$$

So, we converted the repeating decimal into a fraction.

$$0.45454545 \dots = \frac{5}{11}$$