Using Equations in Problem Solving

Strategy for Solving Word Problems

- Read the word problem multiple times;
- Draw a diagram if needed;
- Use x (or any other variable) to represent the unknown quantity, then build the rest of the quantities in terms of x;
- Write the equation and solve it;
- Check the solution in the equation and well as in the original problem.

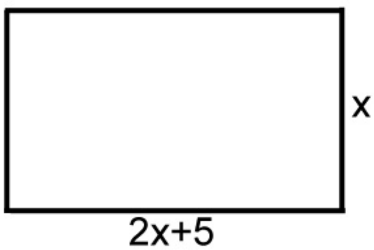
Examples of Word Problems

Problem 1 – Geometric Problem

A rectangular garden has a perimeter of 82 feet. Its length is 5 feet more than twice the width. Find the length and the width of the of the garden.

Solution

The width of the garden is unknown, so we will represent it by x. Then the length will be $2x + 5$.



We will use the perimeter formula $2L + 2W = P$.
In this formula:

Length	Width	Perimeter
$L = 2x + 5$	$W = x$	$P = 82$

Replace these expressions in the formula and solve for x.

$$2L + 2W = P$$

$$2(2x + 5) + 2x = 82$$

$$4x + 10 + 2x = 82$$

$$6x + 10 = 82$$

$$-10 \quad -10$$

$$6x = 72$$

$$\frac{6x}{6} = \frac{72}{6}$$

$$x = 12$$

Thus,

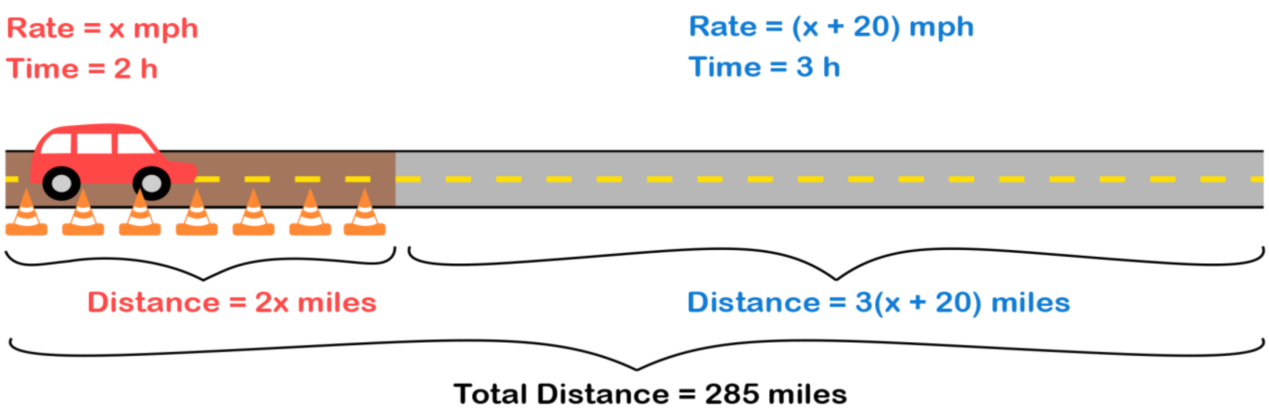
$$Width = 12\ feet$$

$$Length = 2 \cdot 12 + 5 = 29\ feet$$

Problem 2 – Uniform Motion Problem

Brian drove through a road construction zone for 2 hours. When the construction zone ended, he increased his speed by 20 mph and drove for another 3 hours. He drove 285 miles all together. What was his speed in the road construction zone?

Solution



The speed in the road construction zone is unknown, so we will represent it by x.
We will use the distance formula $Rate \cdot Time = Distance$.

	$Rate$	$Time$	$Rate \cdot Time = Distance$
Road Construction Zone	$x\ mph$	$2\ hours$	$2x\ miles$
Clear Road	$(x+20)mph$	$3\ hours$	$3(x+20)\ miles$

Adding together the Distance traveled in Road Construction Zone and the Distance on Clear Road will give us the Total Distance.

$$2x + 3(x + 20) = 285$$

$$2x + 3x + 60 = 285$$

$$5x + 60 = 285$$

$$-60 \quad -60$$

$$5x = 225$$

$$\frac{5x}{5} = \frac{225}{5}$$

$$x = 45$$

Thus, the rate in the road construction zone is $45\ mph$.

Problem 3 – Cost Problem

1,874 tickets were sold at an amusement park for a total of $21,356. If each child paid $9 and each adult paid $14, how many children bought tickets?

Solution

Let x be the number of children, and $1{,}874 - x$ be the number of adults.

	$Number$	$Price\ per\ Ticket$	$Total\ Cost$
Children	x	$\$9$	$9x$
Adults	$1{,}874 - x$	$\$14$	$14(1{,}874 - x)$

Adding together the cost of children's tickets
and the cost of adults' tickets will give us the total cost of $21,356.

$$9x + 14(1{,}874 - x) = 21{,}356$$

$$9x + 26{,}236 - 14x = 21{,}356$$

$$-5x + 26{,}236 = 21{,}356$$

$$-5x + 26{,}236 = 21{,}356$$

$$-26{,}236 \qquad - 26{,}236$$

$$-5x = -4{,}880$$

$$\frac{-5x}{-5} = \frac{-4{,}880}{-5}$$

$$x = 976$$

$$1{,}874 - 976 = 898$$

976 children and 898 adults bought tickets.

Problem 4 – Cost Problem

A grocer sells two types of apples, Fuji and Honeycrisp. One pound of Fuji costs \$2, and one pound of Honeycrisp costs \$3.25. If the grocer sold 10 pounds of apples for a total of \$25, how many pounds of each type were sold?

Solution

Let x be the number of pounds of Fuji apples, and $10 - x$ pounds of Honeycrisp apples.

	$Number$ of Pounds	$Price\ per\ Pound$	$Total\ Cost$
Fuji Apples	x	$\$2$	$2x$
Honeycrisp Apples	$10 - x$	$\$3.25$	$3.25(10 - x)$

Adding together the cost of Fuji apples
and the cost of Honeycrisp apples will give us the total cost of \$25.

$$2x + 3.25(10 - x) = 25$$

$$2x + 32.5 - 3.25x = 25$$

$$-1.25x + 32.5 = 25$$

$$-32.5 \qquad -32.5$$

$$-1.25x = -7.5$$

$$\frac{-1.25x}{-1.25} = \frac{-7.5}{-1.25}$$

$$x = 6$$

$$10 - 6 = 4$$

6 pounds of Fuji apples and 4 pounds of Honeycrisp apples must be sold.

Problem 5 – Investment Problem

A man invested into two accounts, one paying 7% interest per year and the other paying 9% interest per year. He invested three times as much money at 7% than he invested at 9%. His annual interest is $\$3,600$. Determine the amount of money he invested in each account.

Solution

Let x be the amount of dollars invested at 9%, and $3x$ the amount of dollars invested at 7%.

	Amount in Dollars	Amount of Interest	Total interest
At 7%	$3x$	$0.07(3x)$	$\$3,600$
At 9%	x	$0.09x$	

Adding together the amount of interest from each account will give us the total interest of $\$3,600$.

$$0.07(3x) + 0.09x = 3,600$$

$$0.21x + 0.09x = 3,600$$

$$0.3x = 3,600$$

$$\frac{0.3x}{0.3} = \frac{3,600}{0.3}$$

$$x = 12,000$$

$$3x = 3 \cdot 12,000 = 36,000$$

The man invested $\$36,000$ and 7%, and $\$12,000$ at 9%.

Problem 6 – Cost Problem

Anita spent \$60 and bought a dress, a purse, and a tea cup. The purse is twice as expensive as the tea cup, and the dress is as expensive as the sum of the other two. How much does each item cost?

Solution

	The Cost for Each Item
	$2x$
	x
	$2x + x$

Adding together the cost of each item, will give us the total cost of \$60.

$$2x + x + (2x + x) = 60$$

$$6x = 60$$

$$\frac{6x}{6} = \frac{60}{6}$$

$$x = 10$$

The cup costs \$10.

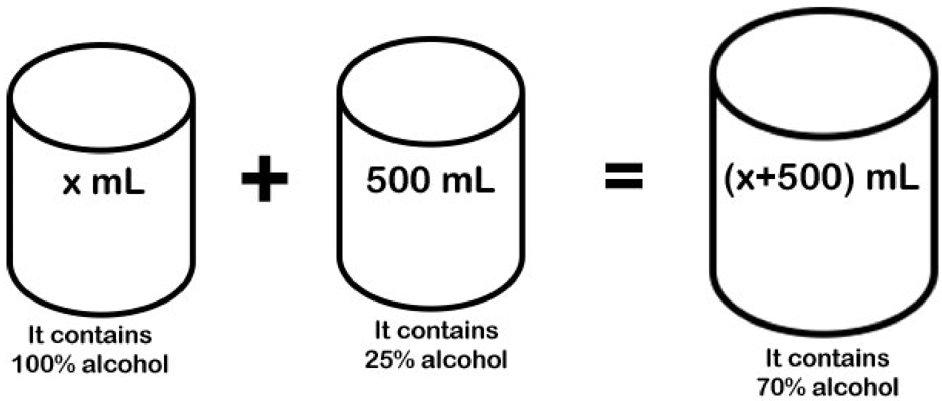
The purse costs \$20.

The dress costs \$30.

Problem 7 – Mixture Problem

John needs to mix two solutions. How much pure alcohol must he add to 500 mL of a solution that is 25% alcohol to make a solution that is 70% alcohol?

Solution



$100\% = 1$ The first container has $1x\ mL$ of alcohol.	$25\% = 0.25$ The second container has $0.25 \cdot 500\ mL$ of alcohol.	$70\% = 0.7$ The resulting container has $0.7(x + 500)\ mL$ of alcohol.

Adding together the amount of alcohol in the first two containers will give us the amount of alcohol in the resulting container.

$$1x + 0.25 \cdot 500 = 0.7(x + 500)$$

$$x + 125 = 0.7x + 350$$

$$-0.7x \qquad -0.7x$$

$$0.3x + 125 = 350$$

$$-125 \quad -125$$

$$0.3x = 225$$

$$\frac{0.3x}{0.3} = \frac{225}{0.3}$$

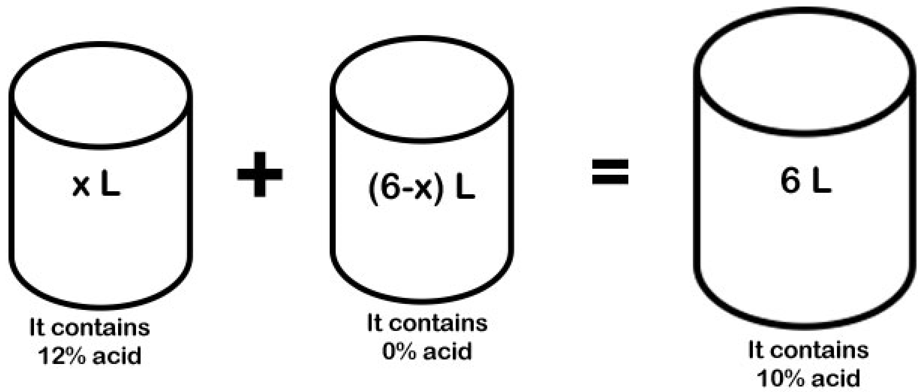
$$x = 750$$

John needs 750 mL of pure alcohol.

Problem 8 – Mixture Problem

Greg would like to mix a solution that is 12% acid with a solution that is 100% water to make $6\ L$ of a solution that is 10% acid. How much of each solution should he use?

Solution



$12\% = 0.12$ The first container has $0.12x\ L$ of acid.	$0\% = 0$ The second container has $0(6-x)\ L$ of acid.	$10\% = 0.1$ The resulting container has $0.1 \cdot 6\ L$ of acid.

Adding together the amount of acid in the first two containers will give us the amount of acid in the resulting container.

$$0.12x + 0(6-x) = 0.1 \cdot 6$$

$$0.12x = 0.6$$

$$\frac{0.12x}{0.12} = \frac{0.6}{0.12}$$

$$x = 5$$

Greg needs $5\ L$ of 12% acid and $1\ L$ on water.